The 6th International Meeting
on Origami
in Science, Mathematics and Education

Program and Abstracts

held at The University of Tokyo
August 10-13, 2014

Edited by Toshikazu Kawasaki, Ryuhei Uehara,
Tomohiro Tachi and Jun Maekawa

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The 6OSME Organizing Committee

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Foreword

Welcome to the 6th OSME in Tokyo. We have walked a long way from the first International Meeting of Origami Science and Technology in 1989.

Going back to those days, there were many scientists who believe that the folding paper has some extraordinary characteristics and abilities hidden within it, and try to discover them. Unfortunately, there were almost no journals to accept their papers and no scientific meetings to discuss them with each other. Professor Humiaki Huzita, a passionate origami scientist and a close friend of mine, finally succeeded to organize the meeting at the house of poet Ariosto in the beautiful city of Ferrara.

After Italy, the sites of our meeting went around the earth. The second in Japan, the third and fourth in US, the fifth in Singapore, and now we come back to Japan, the motherland of origami. We are proud that the series of Book of Proceedings, published through the great efforts of editors, have become now the main sources of information on the subject in these days.

Since the first OSME, both the advance and expansion of this field are far beyond our expectation. The simple symbolic number to tell the situation may be the linear measure. In the 1970~80's, we had imagined the kilometer scale targets for the space applications, say the solar power satellites and solar sails. At present, micro scale targets, say the cell origami, become one of the major fields of research.

Dear scientists, mathematicians, artists, educators, and enthusiasts, please enjoy the meeting at the University of Tokyo.

Koryo Miura,
President, 6OSME

Preface

This volume contains abstracts of all papers presented at 6OSME: The 6th International Meeting on Origami in Science, Mathematics and Education, held on August 10-13, 2014 in Tokyo. There have been 168 submissions, all of which were reviewed by at least two Program Committee members. The Program Committee decided on accepting 137 papers, including twelve poster presentations. We the Program Committee members are happy to announce that two invited presentations are included among the presentations at the conference.

First of all, we are grateful to the authors of the papers for contributing to 6OSME. Without their contributions, the Meeting could not have been possible.

We also gratefully acknowledge the Program Committee members, Roger Alperin, Hideaki Azuma, Erik D. Demaine, Martin L. Demaine, Emma Frigerio, Tomoko Fuse, Koshiro Hatori, Thomas Hull, Yves Klett, Kaori Kuribayashi-Shigetomi, Robert J. Lang, Jun Maekawa, Yoshinobu Miyamoto, Kazuya Saito, Koichi Tateishi, Arnold Tubis, Patsy Wang-Iverson, and Zhong You, for their taking time for reviewing abstracts. Thanks go to the members for their continuous efforts for the Meeting.

Among all people who helped us, we would especially like to express our gratitude to Patsy Wang-Iverson, for her tremendous amount of work, helpful pieces of advice, and continuous support for 6OSME, without which it could have not been possible for us to organize the Meeting this smoothly.

Toshikazu Kawasaki
6OSME Program Committee Chair

Ryuhei Uehara
Tomohiro Tachi
6OSME Program Committee Vice Chairs

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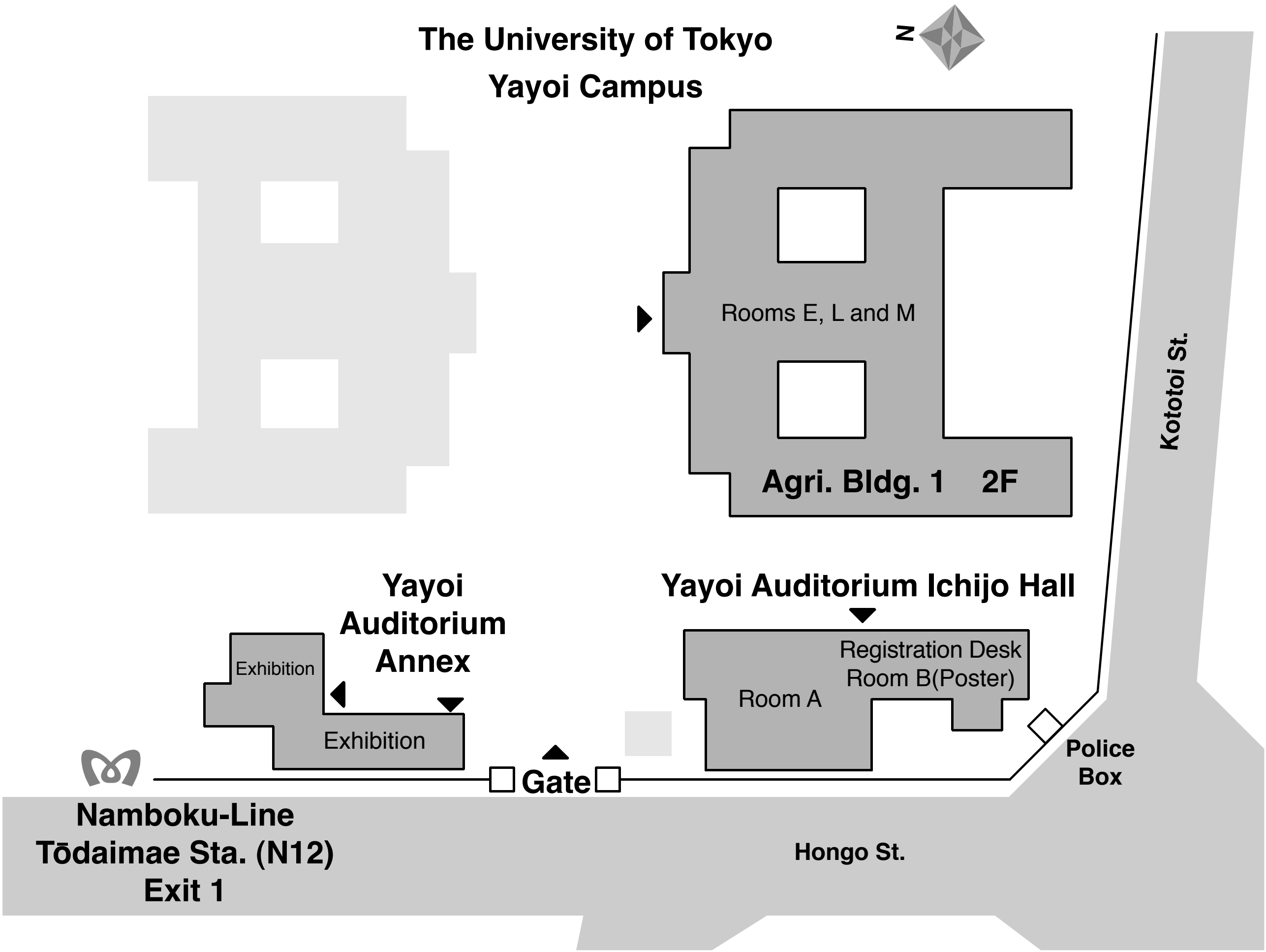

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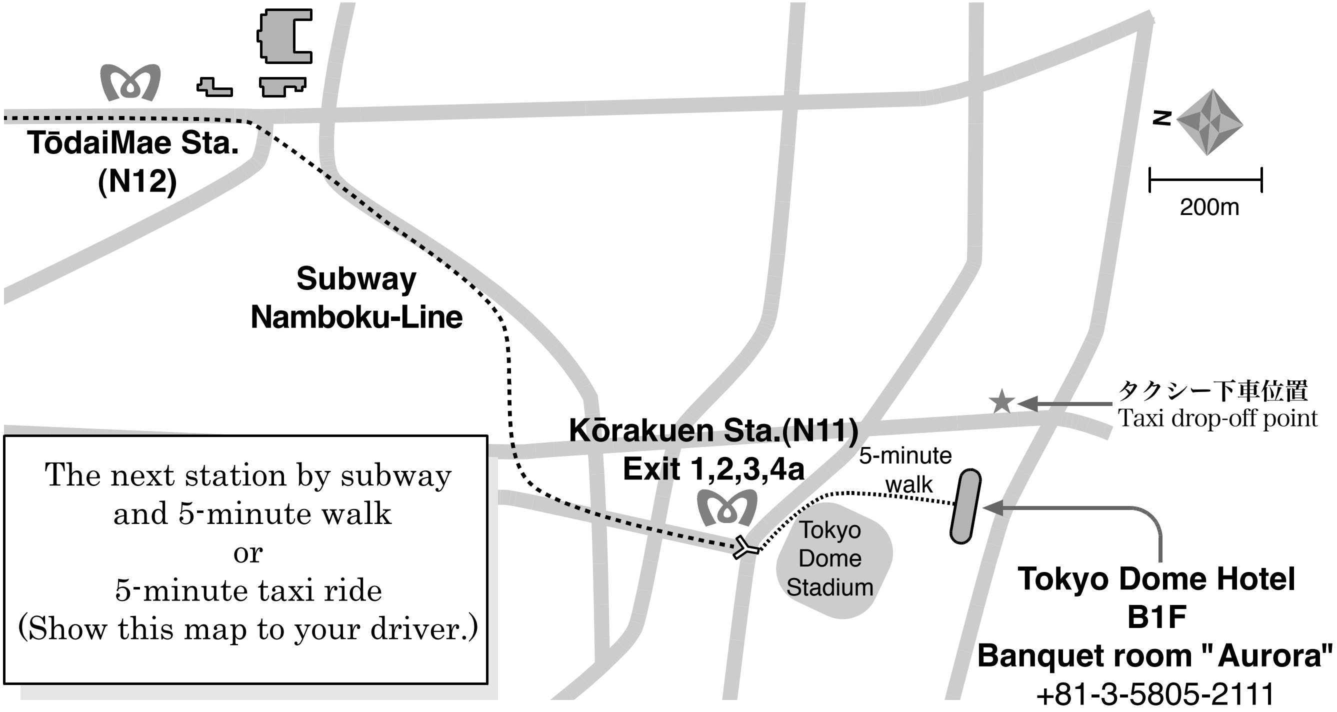

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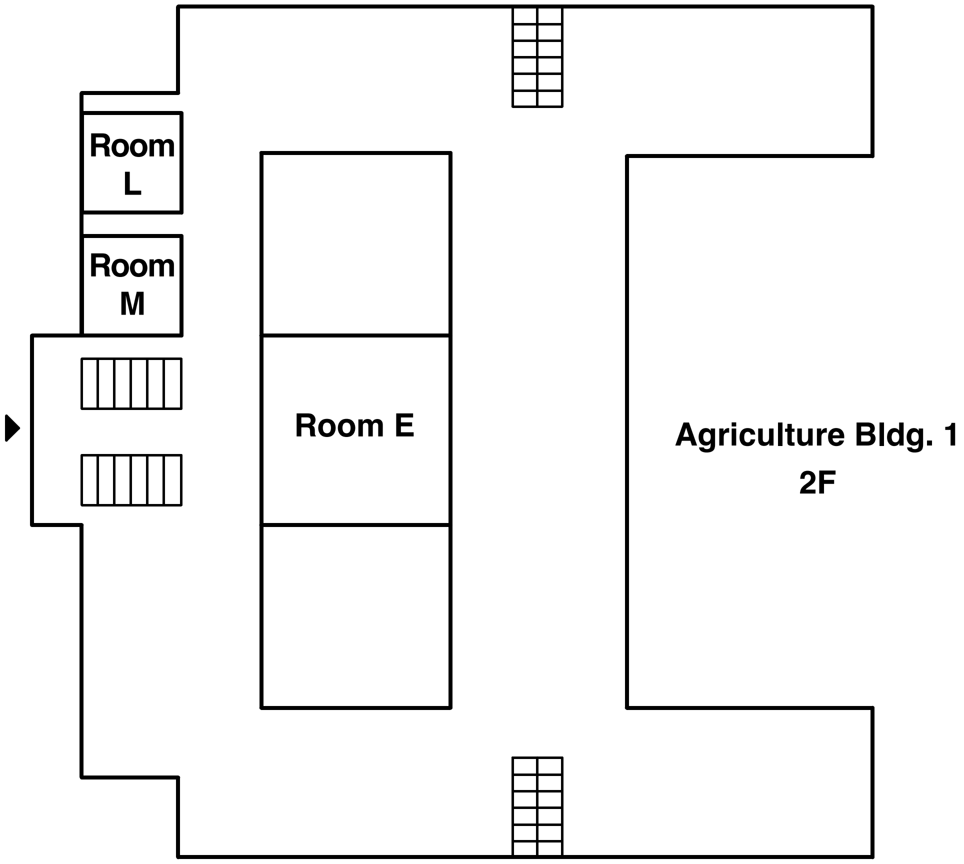
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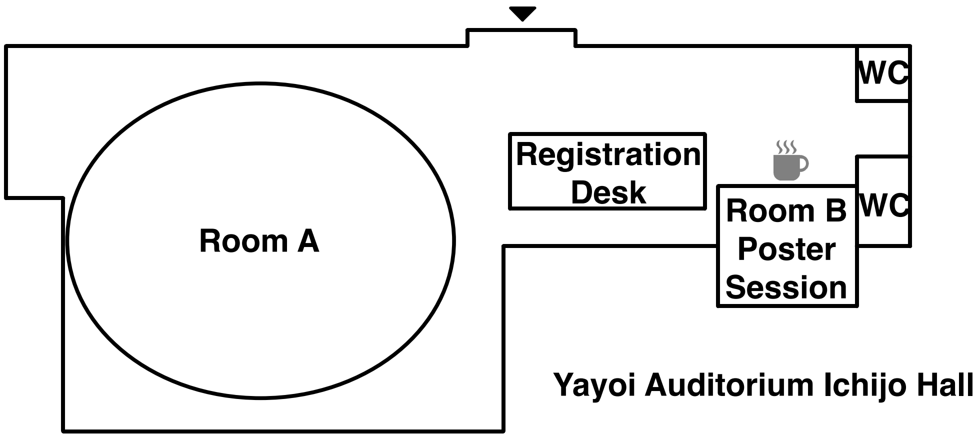
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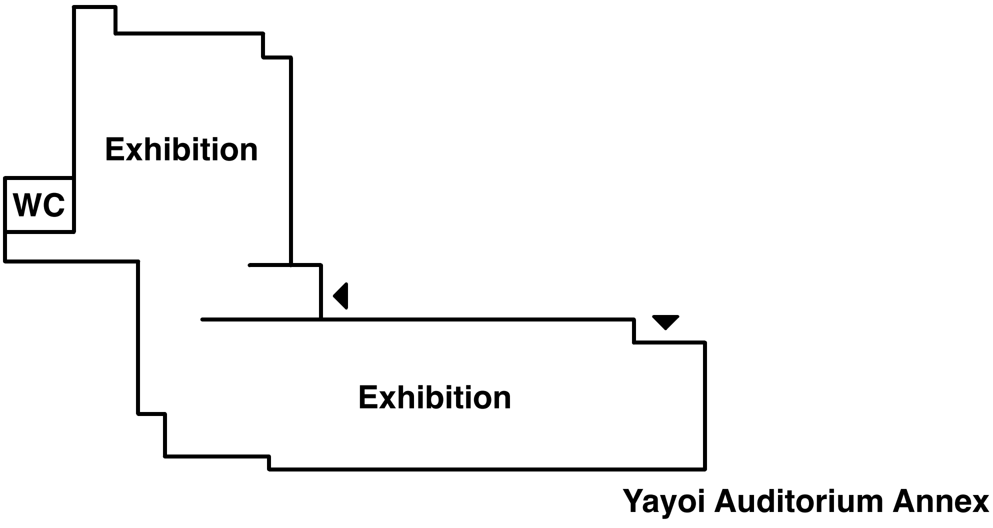
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Timetable

The numbers appearing in the session are submission numbers.

10 sun
Registration
Welcome reception

	11 mon							
Room	A		E		L		M	
900	A0 Opening Ceremony Keynote speech							
1040	break		break		break		break	
1105	A1	Software 87 126 96	E1	Self Folding1 94 183 135	L1	Dynamics 166 161 77	M1	Education 1 52 24 78
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1400	A2	Self Folding 2 31 106 65 91	E2	Art1 102 128 37 62	L2	Combinatorics 36 41 73 84	M2	Education 2 12 9 45 59
1540	break		break		break		break	
1605	A3	Rigid Folding 1 100 44 86 90 98	E3	Design 1 10 103 130 71 155	L3	Math 1 15 123 132 104 107	M3	Education 3 17 121 179 153 67
1805								

Timetable

The numbers appearing in the session are submission numbers.

	12 tue			
	A	E	L	M
900	A4 Keynote speech			
1040	break	break	break	break
1105	A5 Curved Folding 97 164 168	E5 Architecture 43 57 125	L5 Art2 18 64 112	M5 Education 4 66 113 138
1220	Lunch & Poster 14 38 51 53 54 70 92 122 143 152 162 178			
1425	A6 Self Folding 3 48 116 167 147	E6 Pop-Up 1 27 72 115 157	L6 Infinity 40 63 29 55	M6 Rigid Folding 2 156 171 140
1605				
1800	Conference Dinner			

	13 wed			
	A	E	L	M
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1040	break	break	break	break
1105	A8 Design 2 68 165 81	E8 Hinge 28 150 23	L8 Precision 136 170 154	M8 Math 2 105 134 184
1220	Lunch			
1400	A9 Construction 124 158 83	E9 Tessellation 1 82 99 127	L9 Culture & History 35 111 176	M9 Art 3 145 146 131
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1540	A10 Composite Structure 2 61 26 33	E10 Tessellation 2 110 101 142	L10 Pop-Up 2 160 46 47	M10 Art 4 163 133 117
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1745				

6OSME PROGRAM

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Origami works that Kuzuhara Kōtō brought down to us

OKAMURA Masao

Kuzuhara Kōtō was born in Hiroshima in 1812. He suffered smallpox and lost his eyesight when he was three years old. But he left tremendous achievements as a professional teacher of “koto” and “syamisen,” the occupation authorized only for the blind men by the government at that time. In addition, he was regarded as the top-ranking player in the western Japan.

He is also known to have kept a diary. He did not write it in braille but stamped it with wooden types of hiragana (syllabary). He developed his own way to group the types, which he purchased on the market, using his original frames, to align characters with even spacing and lines. It is surprising that he kept the diary for over 40 years even though he himself was not able to read it.

In his diary fragmentary found are the descriptions about origami. He was indeed a great origami folder. He is considered to have learned origami while he was training music playing in Kyoto for several years since he was 11 years old. Fortunately, several dozens of his origami works, including insects, birds, persons, and geometrical figures, have been carefully preserved and remain today. His works contain most of the traditional origami models that had been handed down in Kyoto during the 19th century. It is assumed that he folded these works within a short time in anticipation of leaving these models to next generations.

Though the Froebel’s Kindergarten education started in Tokyo more than five years before his death in 1882, no influence of European origami folding can be seen in Kōtō’s origami models. Thus, his origami models can be said as a complete set of Japanese traditional origami works, which even include all the five origami models that were described as “I have heard but do not know well” in Kayara-gusa, a famous source also known as Kan-no-mado. In addition to such traditional models, included are models that seem to be his original, some of which may be unfinished.

Thus, his origami works does have an important and incomparable value in the origami history, even if we leave aside the fact that these were folded by a blind man, and tells us a lot today.

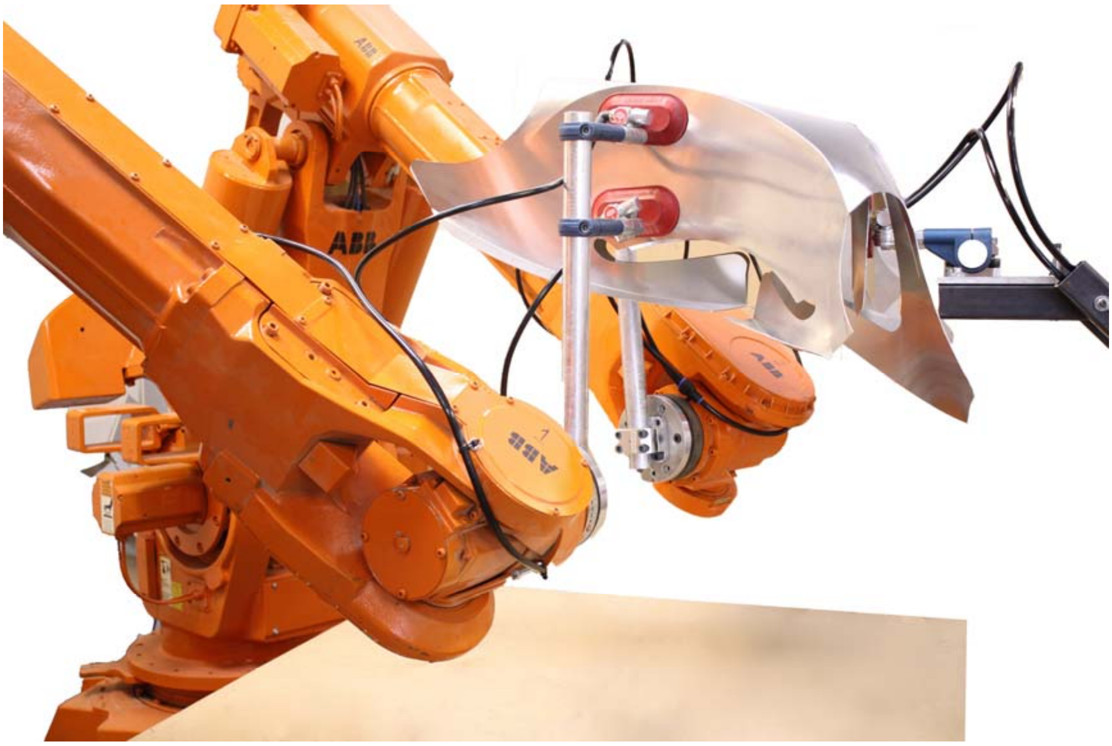
Collection of Kan Chazan Memorial Museum, Fukuyama City
Photographs by KUZUHARA Makoto

Industrial Robotic Folding

Gregory Epps
RoboFold Ltd

RoboFold takes the contemporary origami technique of curved folding and substitutes metal for paper and robots for hands. The result is the breathtaking choreography of sheet metal slowly transforming held aloft by industrial robots. The technology leverages software to transfer the craft technique of hand-folding paper into folding simulations that are used to drive the robotic folding of metal. RoboFold develops a suite of folding, CNC-cutting and 6-axis industrial robot simulation software to enable dataflow through the entire process, enabling each panel to be unique. Applications include automotive, white goods, design and architecture.

The lecture will explore the history and state of the art in computer controlled manufacture, and it's trajectory towards the pervasive use of robotics. Through the analysis of the RoboFold technology, the presentation shows how the application of industrial robots for folding is part of a shift towards robots gaining adoption outside of the highly automated industries such as automotive, and towards a closer relationship and emulation of the craftsman.



Examining TIMSS Items through the Lens of Origami

Patsy Wang-Iverson (G&P Rosenbaum Foundation; pwangiverson@gmail.com)
and Nick Timpone (Primavera PD)

With his invention of kindergarten in 1838, Friedrich Froebel used origami to introduce young children to informal exploration of mathematical concepts. In the early years, his vision and methods were faithfully implemented. Unfortunately, his intent was modified and misinterpreted with the growth and expansion of kindergartens around the world, resulting in the abandonment of the use of origami in instruction in the early grades. Today it is rare to see origami implemented systematically and systemically in kindergarten classes, or, for that matter, in mathematics classes across the grades.

In the 2011 administration of TIMSS (Trends in International Mathematics and Science Study; timss.org), the following grade 4 item was presented to students: *Tom ate half of a cake and Jane ate one quarter of the cake. How much of the cake did they eat altogether* (Item M031297)? Student scores ranged from a high of 84% to a low of 8% across countries; the average score was 23%. Might students introduced to origami in the early grades perform at a higher level?

To examine this question, TIMSS items were administered to students in two classes at grade 8 (one item) and grade 4 (three items), respectively. Students in one class at each grade then participated in an origami activity without discussion of the test items. Three weeks later the items were re-administered to both the test class and control class. This paper will discuss the results from the study and explore how origami may help low performing students improve their understanding and performance while increasing their interest in and enjoyment of mathematics.

Design Art - Taking the Kami out of Origami

by Garibi Ilan

The world of product-design is heavily influenced by Origami. It is prominent in domains like architecture, fashion and furniture design. A closer look shows that classic Origami (a folded surface), is drastically transformed when it is assimilated into a product or a sculpture. Most products imitate an Origami look by applying different techniques such as molding, cropping and welding, but not actual folding.

To make a folded product, many Origami artists choose paper. Origami lamps frequently feature a modular or tessellation design. My Molecules series is such an example, presented in the Design Week in Milano (2011), but even they didn't break the 'paper barrier'.

My main insight was to switch materials. If paper jewelry does not sell, let's move to metal. How can I fold metal? How do I fold ceramics? Can paper models be used as molds for concrete?

These questions set me on a quest for new materials and processes. An example of what I found is Palmas, a series of vases made from concrete cast into paper molds. This is a complex, irreversible process, requiring a thorough grasp of the process of casting and de-molding.

Leather and its by-products can also prove a fertile ground to further products, such as bags and shoes. Leather, being thick, is not foldable without special processing.

Metal is, of course, foldable, being significantly similar to paper, but it is done only after special treatment.

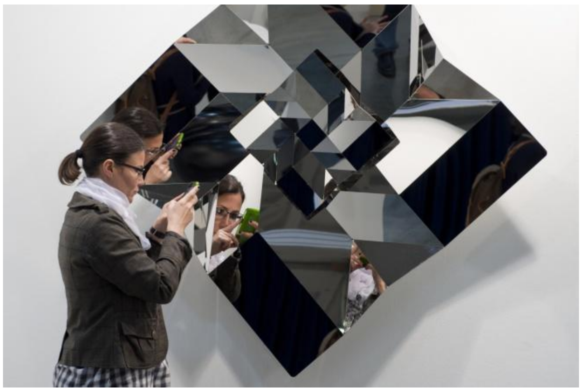
This paper attempts pave a new road for origami artists, showing that new foldable substances can help us in turning them into design products and works of art.

Diamond corrugation vase, Designed by Garibi Ilan

Wall Panel Designed by 3D Surface

96 Molecules lamp, Designed by Garibi Ilan for Aqua Creations

Palmas concrete vases , Designed by Garibi Ilan and Ofir Zucker



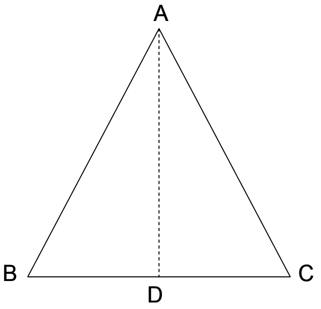
Metal Origami Art "Diamond", Designed by Garibi Ilan

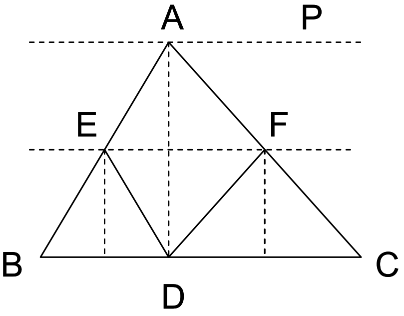
Using Paper Folding to Solve Problems in School Geometry

Huang Yanping, Southwest University, Chongqing, China
Lee Peng-Yee, formerly National Institute of Education, Singapore email: pengyee.lee@nie.edu.sg

In this presentation, we shall demonstrate how problems in school geometry can be solved by means of paper folding. We assume as given six paper folding methods. Then we proceed to prove all other properties concerning parallel lines, triangles, quadrilaterals and circles. We give two examples here, and refer to the full paper for more details.

Example 1. Given an isosceles triangle ABC with $AB = AC$, fold an angle bisector which bisects the vertex angle, bisects the base, and is perpendicular to the base.
Solution. One folding method says that we can fold a given line such that it coincides with another given line. So we can fold AB onto AC. By symmetry, all the properties follow. See left figure below.





Note that using the given folding methods, we can prove the so-called parallel lemma: “Given two parallel lines, fold one onto another. Then the folding line cuts any line segment connecting the two parallel lines into two equal halves.”
Example 2. In a triangle ABC, the midpoint of AB is E and the midpoint of AC is F. Verify by paper folding that EF is parallel to BC and the length EF is half of that of BC.
Solution. By one folding method, we can fold a line passing through A and perpendicular to BC. Also, we can fold another line passing through A and parallel to BC. Call it AP, and AP is parallel to BC. So we can fold AP onto BC. Then apply the parallel lemma, E and F being mid-points lie on the folding line. Hence EF is parallel to BC. See right figure above. It follows from Example 1 that the length of EF is half of that of BC.
The above example is known as the mid-line theorem. The key step is applying the parallel lemma after folding AP onto BC.

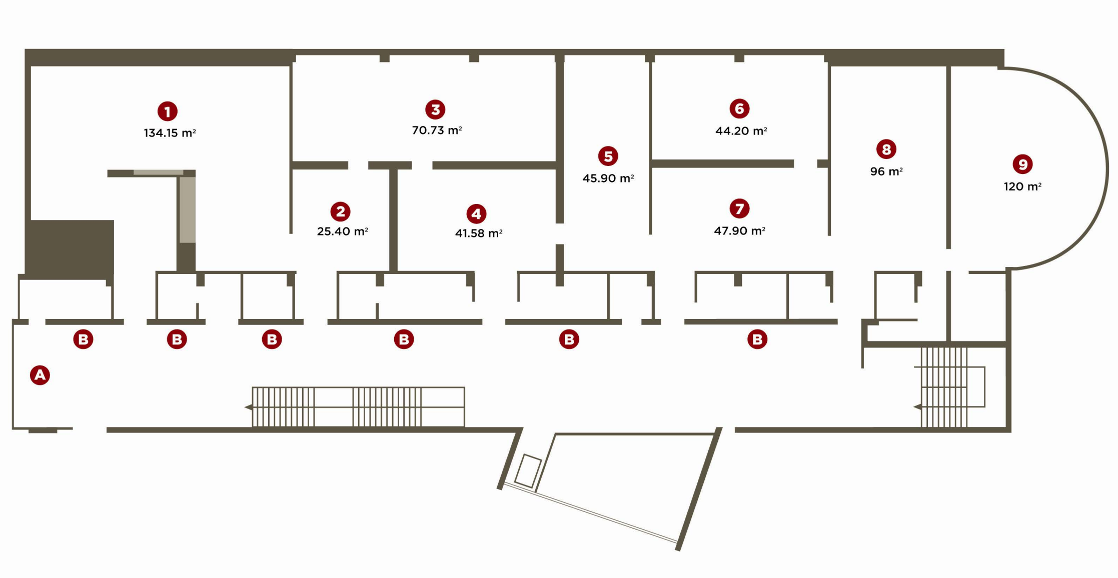
In the full paper, we list the six paper folding methods, which are known as axioms in the literature of origami. Then using the technique as shown in the above examples, we prove the standard properties of parallel lines, triangles and others in school geometry. The materials are developed for teacher training. More materials can be found in a book written in Chinese by the two authors. The book is titled *Paper Folding and Mathematics* and published by Science Press, Beijing, in 2012.

EMOZ (Educational Museum Origami Zaragoza)

18th December 2013 was the inauguration of EMOZ, the first Origami Museum in Europe, where Origami is treated as art.

This museum is at Centro Historias, Zaragoza (Spain) in a building of the city council and managed by an association (Origami Zaragoza), the oldest origami group, founded in 1944

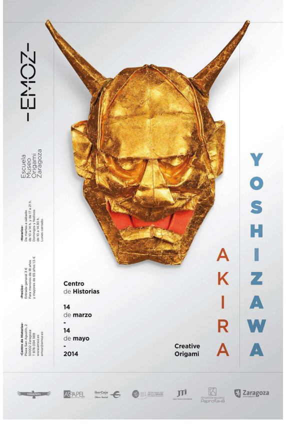
More than 800 Square meters, divided in Shop, Exhibition, Workshop/Library
The exhibitions of the Museum change every 2-3 months



The first 2 temporal exhibitions at the museum were

Victor Coeurjoly
18th December 2013 to 5th March 2014

Akira Yoshizawa
14th March to 14th May 2014



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Axioms for Origami and Compass Constructions

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We describe the axioms of a single fold origami system where, in addition to the usual Huzita-Justin axioms, one may also use a compass to create circles. An axiom of this type was implicitly used by [ES'01], which provided as fold lines the common tangents to a circle and parabola. This allows one to construct the roots to the general quartic polynomial equation.

Recently, this idea of constructions using a circle together with origami has been pursued by [KGI'11]. They added three axioms to the usual Huzita-Justin axioms for single fold origami, obtaining a system which also is *not* more powerful than single fold origami.

In this article we complete these systems by considering the full range of axioms for constructions with compass and single fold origami. We allow foldings which align any of the geometric objects, either point, line or circle with another such object by reflection across the corresponding fold, For us aligning will mean incidence when a point is involved or in the case of two curves the alignment is by tangency.

There are 22 axioms in all; the new axioms labelled O_{5b}, O_{6b}, O_{7d} are those of [KGI'11]. Although our system is not more powerful than ordinary origami, it does allow one to elegantly fold the tangents to a given conic from a given point or the common tangents to two conics. We give two additional methods for trisection of an angle which are similar to ancient techniques based on *neusis* and also a method for construction of cube roots.

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Origami-Inspired Deductive Threads in Pre-Geometry

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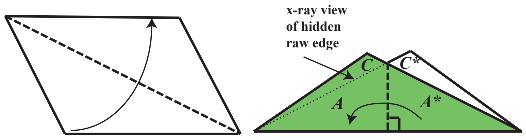
Among the many virtues of origami in the informal- or pre-geometry part of early mathematics education are its empowerment of students to: 1) do hands-on (versus computer-aided) constructions such as line/angle bisection, perpendicular and parallel lines, golden sections, etc. which require substantially more steps when using a straight edge and compass; and 2) verify the congruence of line segments, angles, and polygons by folding these elements on top of one another (see, e.g., Coad, 2006). Its pedagogical value is in part associated with the synergy of using it both to illustrate and apply mathematical concepts as well as to fold a variety of "interesting" forms (Cornelius and Tubis, 2009).

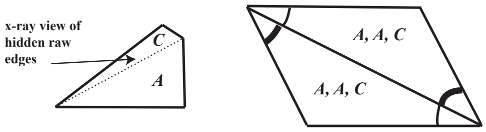
In this paper, we explore one of the many possible ways of using origami as part of a gentle and engaging introduction to deductive threads in pre- (Euclidean) geometry. We introduce: 1) four key planar-geometry assertions or postulates about single, perpendicular, and parallel lines that are strongly suggested/supported by easy folding explorations; and 2) the assertion of congruence of two triangles separated by a diagonal of a general parallelogram (also simply demonstrated by folding, as shown in Figure 1). We then show that these assertions can be used to infer a substantial body of results, *all of which can be made all the more vivid and memorable by being also easily demonstrated by folding.*

Because we only deal with geometric assertions that are easily motivated by folding, and inferences that are easily corroborated by folding, the logical threads are somewhat idiosyncratic and limited. We e.g. *do not* attempt to mimic the way that the sufficiency conditions for triangle congruence (angle, side, angle; side, angle, side; side, angle, angle; side, side, side) are used in higher geometry. Nevertheless the inferred results (including the Pythagorean Theorem and its inverse; properties of parallel lines and their transversals; and area and angle properties of triangles, rectangles, parallelograms, and trapezoids) constitute a substantial body of practical knowledge that should provide a solid background for subsequent higher-level studies of geometry.



1. Make a patty paper parallelogram, and valley fold to give two triangles.

2. The front (green) triangle has sections *A*, *A**, and *C*. The rear one has section *C** and sections congruent to *A* and *A**. Valley fold.



3. The *A* and *A** sections are seen to be congruent, as are the *C* and *C** ones. Unfold completely.

4. The congruency of the two triangles implies that each of the two pairs of alternate interior angles are congruent, that the opposite sides of a parallelogram are equal in length, and that the area of each triangle is half the area of the parallelogram.

Figure 1. Folding verification of the congruence of the two triangles separated by a diagonal of a parallelogram. The two congruent triangles designated ***A, A, C*** in step 4 are each constituted as the green triangle ***A, A*, C*** (= ***A, A, C***) in step 2.

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Design of Closed Origami Polyhedral Surface by Straight-crease Couplets

Cheng Chit Leong

Abstract

The surface of an object is a continuum of different curvatures. These curvatures, generally, are not developable. We may, however, represent the surface by facets or polygons that are adjacent to one another. For practical reasons, the number of polygons need not be large, but it should be sufficiently enough to present the surface in a realistic and aesthetic manner.

In flat folding origami, the surface is essentially defined by complete polygons and by edge determined (imaginary) polygons. The objective of this paper, however, is to present a method for designing and folding a 3-dimensional closed polyhedral surface representing the object from an uncut sheet of paper.

Several methods for designing and folding polyhedral models based on triangulation and tucking molecules have been written and presented. These include that of Tachi's which defines a method for designing polyhedral models based on tucking molecules.

This paper presents a method which differs from the others in several fundamental aspects.

Overall, an aesthetic (judgmental) approach of defining the polygons representing the surface is adopted. It moves away from triangulation and attempts to include polygons of more than three sides. The method takes into account that several polygons are already joined at the start. This immensely minimizes the number and reduces the complexity of edge and apex tucking molecules in the other methods.

The key to the approach adopted is the use of couplets to align all the polygons together. Each couplet consists of a straight mountain crease and a straight valley crease, with an overlapping section. The couplets may serve as edge tucking molecules but may also over run the surface of polygons.

An example of designing and folding the face of the bust of Nefertiti is given, from pictures of the object. A front and a side of view of the object are sufficient to develop the polygons necessary for its representation. The true shape and size of the polygons are obtained by geometrical drawing. The polygons are then arranged such that folding by couplets can be accomplished.

Topological Origami Models of Non-Convex Polyhedra

Andrea Hawksley[*]

Abstract. We present a series of 'topological' modular origami models of star polyhedra, as well as a method for creating such models and a theoretical enumeration of which models are possible to create with our methodology.

1 Introduction and Previous Work. Modular origami is a standard technique for creating polyhedral models, but most existing. Most current origami models of nonconvex polyhedra obscure the underlying connectedness of their faces. We present a series of 'topological' origami modular constructions that emphasize the internal connectedness of self-intersecting polyhedra. The models bear some resemblance to other modular origami Lang's 'polypolyhedra', as well as to the work by Hideaki Kawashima, in that they show underlying intersecting geometries; however their work is focused on creating edge skeletons of the shapes, while the current work focuses on representing the polyhedral faces [3, 4]. Each model is not of intersecting compounds, but a single self-intersecting star polyhedron. Sculptures similar to those created with our technique have, however, appeared in non-origami sculpture [1, 2, 5, 6]

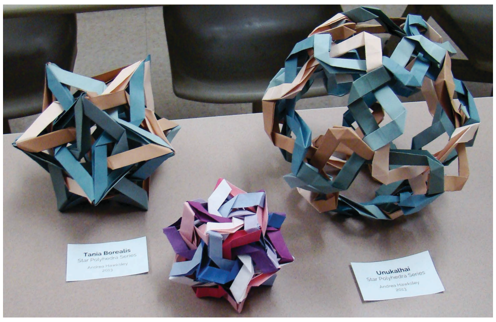

Figure 1: Origami models of the great rhombic triacontahedron, the medial rhombic triacontahedron, and the small triambic icosahedron

2 Constraints. We use the constraint that models must be made out of identical units or modules that represent faces rather than edges. We choose to connect the modules at their vertices, a choice that best shows the underlying connectedness of the faces. This requirement makes it difficult to model polyhedra with nonconvex vertex figures. With the constraint that all vertices be represented in the figure and that each module represent a strict subset of a face, none of the Kepler-Poinsot polyhedra are possible. Allowing models where not all vertices are represented enables us to theoretically model 33 dual nonconvex uniform polyhedra, after discarding those with vertices at infinity and those with self-intersecting faces.

Figure 2: A single module of the great rhombic triacontahedron model

The underlying geometries of these 'topological' models are constructed by removing parts of each face of the nonconvex polyhedra symmetrically such that the remaining portions of each face no longer intersect. The modules and the resulting models are necessarily chiral.

Figure 3 shows an example of a face of a great rhombic triacontahedron. In order to maintain connectedness while avoiding intersection with other units, each face must be represented by a rotationally symmetric unit that passes through all of the green points (where the units connect and the center point) but avoids all of the red points (where more than two faces intersect in the original polyhedron). These constraints still allow room for much artistic interpretation, so two models designed with this technique and based on the same polyhedron may still be very visually distinct.

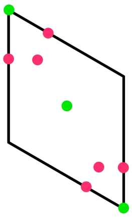

Figure 3: Face of the great rhombic triacontahedron, with intersection points marked

3 Future Work. Future work in the area of topological origami models include models of non-isohedral polyhedral by using non-identical modules or by not representing every face, models with self-intersecting faces with modules that are not strict subsets of the underlying faces, and models of polyhedra with nonconvex vertex figures by connecting faces along edges rather than at vertices or by connecting faces in a non-convex manner, perhaps by removing parts of each face at the vertices.

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Application of Rigid-Foldability Condition to Yield Line Analysis

Watanabe Naohiko

Yield line analysis has been used for predicting the ultimate strength of a structure, e.g., local buckling and lateral buckling, of a thin-walled member. This study applies the rigid-foldability condition to yield line analysis for the calculation of the internal work around yield lines; this approach was validated through its application to the prediction of the static compressive strength of steel square pipes.

The concept of the method for obtaining a decay curve by the yield line analysis is explained as follows.

Consider the equilibrium of work by an external force when axial displacement δ occurs and the internal work by folding around hinge lines through an assumed collapse mechanism. By differentiating the equation of this equilibrium, we obtain the following equation:

$$\mathrm{N} = \sum \frac{dW_H}{d\delta}$$

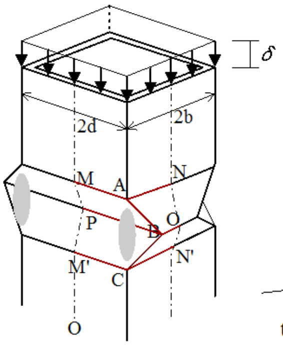
where N is the axial force, δ is the axial deformation of the member, and W_H is the internal work around the yield lines. From this equation, it is evident that when the infinitesimal displacement $d\delta$ is given, the axial force N is obtained by dividing the increment of internal work around yield lines, dW_H, by $d\delta$. Further, the axial force N is obtained sequentially by increasing the axial displacement δ; then, the decay curve can be obtained. The curve under the collapse mechanism for which the lowest ultimate strength is obtained is considered as the true decay curve. Here, the increment dW_H of the internal work around yield lines is determined by the strain quantity and the volume of the strain fields in the plate, which are functions of the rotational angles around these lines and the lengths of the hinge lines, respectively.

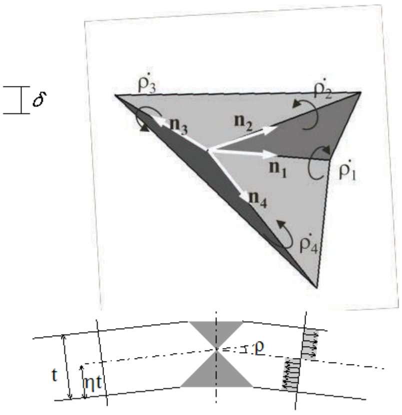
Under the assumption that each angle between two hinges around the vertex-concentrated hinge lines is invariant, the mode of the rotational angular velocity around the hinge lines can be obtained using the rigid-foldability condition. The rigid-foldability condition for the vertex-concentrated hinge lines is expressed as follows:

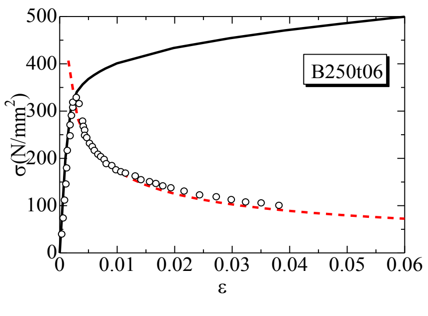
$$\sum \dot{\rho}_i \boldsymbol{n}_i = \mathbf{0}$$

where $\boldsymbol{n}_i$ denotes the direction of each hinge line and the combination $\dot{\rho}_i$ indicates the allowable mode of the rotational angular velocity. The value of increment of internal work can be obtained from the change amount of the rotational angle.

Finally, the developed method is validated through a comparison between the ultimate strength obtained by it and previously obtained experimental results.







Origami as Teaching Media for Early Childhood Education in Indonesia
(Training for Teachers)

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In this paper, the author will describe the experiences in developing origami as one of the teaching media for early childhood education in Indonesia.

Origami can be used as learning media in the school. Easy and cheaper material for origami activities makes it an attractive method for teaching children. Other benefits are for the children motoric abilities, development of child's brain, and introduction of mathematical term, especially geometric verb.

At the present, early childhood schools in Indonesia are increasing rapidly. Origami as one of the teaching media for education is not yet known well by teachers in Indonesia and also not yet fully adopted in the education curriculum.

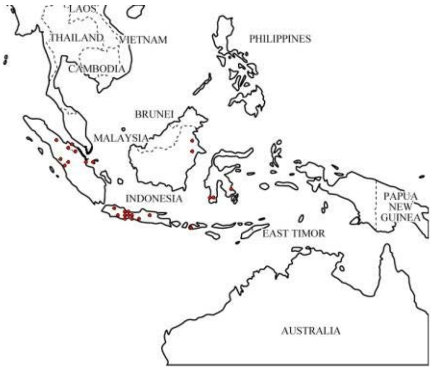
As an origami pioneer and master student in the Indonesia University of Education, the author has an objective to develop origami as a teaching medium in the school, especially for early childhood education. One of the activities under this objective is to provide training and workshop for school teachers.

The material that will be provided during training for origami as a teaching medium include:

- What's origami?
- Benefits from using Origami as a teaching medium
- Basic origami techniques (origami symbols, basic folding)
- Origami models for children's level
- Tips for teaching origami to children

The author collaborated with the teachers association to hold the workshop and training in Indonesia. Since 2006, the author has conducted more than 50 workshops in Indonesia with varying numbers of participants (25 to more than 520 persons) and different areas (from big cities to remote regions). Details of the activities for training can be seen at: www.origami-indonesia.com. Some early childhood schools now use origami as one of the teaching media, which is included in their local curriculum.

The wide area of Indonesia, the diversity of the people and economic condition are challenge fors the author to introduce and develop origami in Indonesia as one of the media for education.



Note : The red dots show the places that have been conducted workshop

Improvement and optimisation of non-Miura foldcores under impact loading

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Foldcores are sandwich panels formed by bonding skins to a partially folded origami or kirigami core pattern. The mechanical properties of Miura-base foldcores are extensively studied in literature, however several non-Miura origami and kirigami core geometries have been proposed as alternatives. This paper presents a continuation of previous work by the authors towards the assessment of the mechanical behaviours of such cores relative to existing Miura-type foldcores.

First, recent experimental results are compared for two curved-crease foldcores, two kirigami foldcores, and two modified kirigami cube foldcores under out-of-plane impact loads. The two latter patterns were generated with geometric modifications known to improve weight-specific energy-absorption capacity, Figure 1. All prototypes were constructed with a sequential folding method with an aluminium material and density comparable to that used in prior studies for Miura-type foldcores. The second curved-crease foldcore compared favourably with a Miura foldcore in terms of strength, with an increase in peak stress of approximately 40%. The second modified cube foldcore compared favourably with a Miura foldcore in terms of energy absorption, with an increase in average crush stress of approximately 90%.

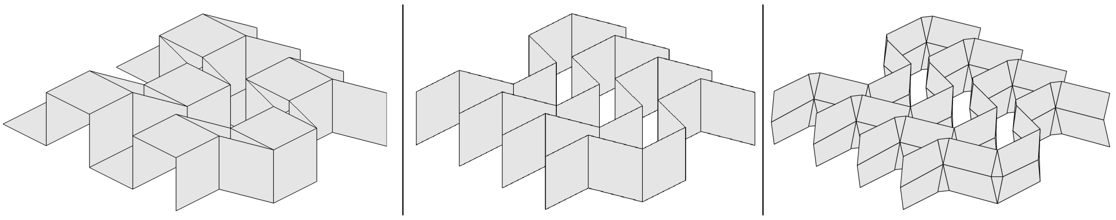

Figure 1. Kirigami cube pattern, left. Modified cube strip pattern, centre; and sub-folded cube strip pattern, right.

A numerical optimisation study was then undertaken on the second curved-crease and the sub-folded cube pattern to assess the extent to which the experimental behaviours could be improved. An optimum CC geometry was achieved which numerically displayed similar strength and energy-absorption to a commercial honeycomb. An optimum cube strip configuration was also numerically found with potentially higher energy absorption than commercial honeycomb. On going work is focused on the construction and testing of the optimum configurations in order to assess the accuracy of the numerical optimisation findings.

Keywords: foldcore; rigid origami; kirigami; quasi-static impact

Design and analysis of morphing folded shell structures

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Recent publications have identified a morphing sandwich panel mechanism that is constructed from layered, rigid-foldable, Miura-ori sheets. Two outer layers are set with a 'face' geometry and a middle sheet with a 'core' geometry, each designed such that global rigid-foldability is preserved over a partial range of motion: from a flat-folded state to a deployed configuration. However, the existing mechanism generates opposing design constraints: hinge location for favourable load distribution behaviour causes unfavourable stiffness behaviour, and vice versa. This paper demonstrates a modified morphing panel arrangement, termed the V2 mechanism, Figure 1, which overcomes this limitation and forms a stiff deployable plate structure.

Initially, two aspects necessary for the design and application of V2 mechanisms are presented. First, core and face geometric conditions required for the new mechanism are identified. Second, construction methods required to translate this idealised mechanism into non-zero thickness engineering structures are demonstrated. An example steel morphing beam is constructed with digitally-fabricated hinge connections.

Figure 1. Single unit of a morphing panel (V2 mechanism), constructed from 1.0mm thick mild steel. Flat-folded, on left; deployed configuration, on right.

Experimental and numerical static analyses are then performed on the steel morphing beam and an equivalent, non-morphing beam. Similarities and differences between their load distribution behaviour and mechanical performance are discussed. Finally, several extensions to the planar V2 mechanism are presented, generated by substituting the Miura-ori morphing core pattern with alternative, Miura-derivative patterns. A radial morphing mechanism is formed from a Tapered Miura core pattern. Single-curved arch and tube mechanisms are formed with a Non-Developable Miura and Conical Miura core patterns, respectively.

Keywords: folded shell structure; rigid origami; Miura-ori; morphing panel; sandwich panel

Self-Intersecting Origami with Cuts

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A number of researches on computer aided origami design and folding simulators have progressed in recent years. Most of these researches assume that the sheet of paper does not stretch and there are no self-intersection. The *no-stretch* condition is easily met by representing origami with a set of rigid polygons or a set of developable patches. However, the *no-self-intersection* condition is rather difficult to be satisfied since it is not avoidable by just considering its local geometry. Therefore, generating folding animations without self-intersection is still a difficult problem. Avoiding intersections in the design stage is also problematic. We usually consider that cutting and gluing are prohibited in origami design. However, adding tiny cuts can make possible, in most cases, the execution of models containing self-intersections using origami. This technique creates a potential for generating completely new origami designs. From the engineering point of view, the cost for cutting is usually much lower than the cost for gluing, which makes the *no-cut* condition easier to ignore. Some forms of paper-crafting, such as origami architecture and *kiriokoshi* are based on this concept.

In this work, we try to restrict the cuts so that they are added just for avoiding physical collisions of sheets as shown in Fig. 1. The restriction also requires the cuts to have no gap, i.e. the hole formed by a cut must have an area equal to zero, both on the unfolded pattern and on the folded shape. In this paper, we show some attractive examples that were created just by adding tiny cuts. For example, it becomes possible to make a torus shape (Fig. 2) from a single rectangular sheet. We propose a method for designing this kind of origami and its crease pattern, exploring the new domain of origami with self-intersection.

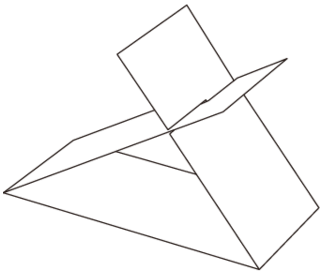

Fig. 1 Simple example of a self-intersecting origami, which has two half width cuts to avoid collision.

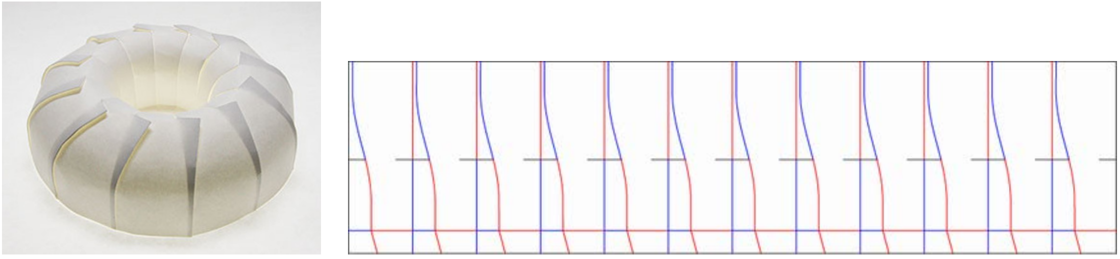

Fig.2 A torus-shaped origami (left), which has self-intersections at the top, and its crease pattern (right).

Foldable Plate Structures with Rolling Joints

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Foldable structures have the ability to transform from a closed or folded configuration to an open or deployed configuration. This paper studies the foldable plate structure based on origami, which consists of a series of plates, connected at their edges by rolling joints, allowing each plate to rotate relative to the common axis of the neighboring plates.

Rigid origami realizes a deployment mechanism with stiff panels, which has advantages for various engineering purposes, especially for designs of retractable roof structures or kinetic architectures, such as shown in Fig.1. In a mathematical context or a paper model, origami is regarded as an ideal zero-thickness panel. However, this is not true when this mechanism is used in engineering practice.

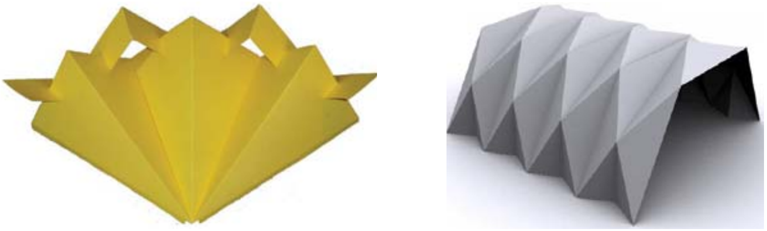
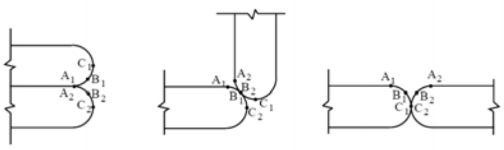
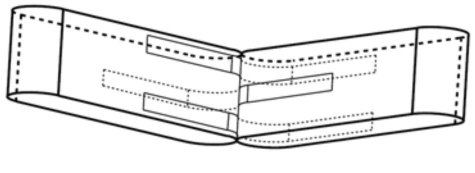


Fig.1 Kinematic architectures

Figure 2 shows a type of connections for the plane linkages. Joint surfaces are cylinders of equal radii in rolling contact. When the linkage moves, one half-cylinder rotates about the other half. Also there is no slide between the two surfaces. Hence the distance between the two cylinder centers is constant. The geometric design of rigid origami with rolling joints was studied in this paper. Due to the avoidance of sliding, it is possible to construct joints having very low coefficients of friction. Firstly, the implementation method of this joint is given. There are many ways to implement the rolling joints. Two methods are given in this paper. One is semi-cylinder connected with tapes, and the other is semi-cylinder connected with cables. Then the mechanical concept of planar four-bar linkages with rolling joints is introduced to describe the topological relationship between rolling joints and the links. Furthermore, the closure equations, which must be satisfied throughout the motion of the linkage, are developed.



(a)Concept of rolling joints



(b)Rolling joints connected with tapes

Figure 2 Rolling joints

Using the Fujimoto Approximation Technique to teach Chaos Theory

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The author has taught high school enrichment mathematics for two decades introducing students to ideas from chaos theory, number theory and algebra. Previous work on the Fujimoto technique has connected it to binary representations of fractions [1] and the orders of elements in modulo arithmetic [2]. This paper describes how chaos theory might be taught to high school students using the Fujimoto approximation technique as a concrete motivational activity.

Each fold (from the left or right) in the Fujimoto technique is represented by the letter L or R. If we denote the left edge of the paper with the number -1, the middle with 0 and the right edge with $+1$ then if x is the location of the current crease the next crease is given by the functions

$$L(x) = \frac{1}{2}(x-1), \qquad R(x) = \frac{1}{2}(x+1),$$

depending on which side the paper is folded from.

Sequences of folds correspond to strings of letters which play the same role as symbolic dynamics in chaos theory [3]. If a cyclic pattern of creases such as $LRRLR$ is repeated indefinitely, then the creases will converge to the solution of the equation $R(L(R(R(L(x))))) = x$. Such cyclic strings correspond to Fujimoto creases occurring at rational fractions of the paper width. Infinite strings which do not repeat converge to creases occuring at irrational subdivisions of the paper.

The cyclic strings and rational numbers can be related to the existence of periodic orbits in the shift and tent maps, which are commonly used when teaching chaos. In my course, I introduce a much simpler function than the shift or tent maps which is given by $V(x) = 2|x| - 1$. This function is topologically conjugate to the tent map and has the same set of cycles as those obtained from the shift map and the tent map and are the exact mirror images of cycles obtained from the Fujimotoi process.

The factors of $\frac{1}{2}$ in the functions L and R cause the Fujimoto process to improve with accuracy and converge to the relevant fractions; whereas the factor of 2 in the function V does precisely the opposite and produces the characteristic sensitivity to initial conditions which is the hall mark of chaos. Thus, these two process can be pecisely identified as being inverse processes (after certain transformatins of variables).

Topological mixing and transitivity of orbits can also be related to questions such as how to efficiently get an approximation to any number starting from any other given number by minimizing the number of Fujimoto folds required.

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Micro-patterned polymer gel multilayers for reversibly self-folding origami

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Origami is a powerful technique for creating three-dimensional structures from two-dimensional sheets, and correspondingly, self-folding materials have attracted considerable interest for the design of actuators and remotely deployable devices. While numerous methods for fabricating self-folding structures have been previously reported, very few have approached the level of complexity associated with traditional origami crease patterns. We describe a simple method to fabricate self-folding origami based on tri-layer films of photo-crosslinkable copolymers. Stresses developed during swelling of a thermally-responsive hydrogel layer bonded to thin rigid polymer layers allows for the definition of micrometer-scale hinges that reversibly fold to well-defined angles, with control of mountain and valley assignments. This method allows for the preparation of arbitrarily complex crease patterns with high fold densities, offering promise for the fabrication of advanced biomedical devices, soft micro-scale robots, and meta-materials with highly tailorable and non-classical mechanical properties.

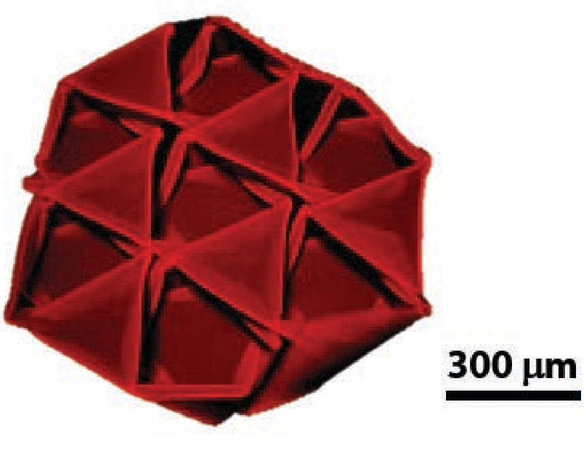

Figure 1. Laser scanning confocal microscopy 3D reconstruction of a self-folded octahedron-tetrahedron truss origami prepared from a hydrogel film with a thickness of 1.5 μm.

Thin-Walled Deployable Grid Structures

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Abstract

This paper concerns with the design of a deployable grid structure which takes up little space when folded away during transport or storage, but can be deployed to provide a useful and structurally rigid three dimensional grid.

The only known square grid is the assembled square grid, formed by sliding together cardboard strips with pre-cut slots, Fig.1. It has already been used in packaging for separating individual products within packing boxes, or as a means of filling excess volume. This type of grid lacks load bearing capacity for the strip is easy to collapse sideways under any significant load.

The new concept reported here was developed using a known flat-folding origami pattern known as the Tachi-Miura polyhedron, Fig. 2. This rigid folding origami that forms a flat quadrilateral grid was combined with a raised web structure in the middle, forming a grid structure, Fig. 3. The raised web structure is compatible with the original folding concept and the new grid structure remains rigidly foldable. Moreover, the new grid structure can be expanded by repetition, leading to the creation of large deployable grid structure shown in Fig. 4. The concept can be readily extended to other geometries including multiple layered gird structures, curved web, and structures with hexagonal grid. We have also demonstrated the concept is valid for material with none-zero thickness. With a suitable selection of materials, these structures can be used as roofs, partition walls and impact resistant packaging materials.

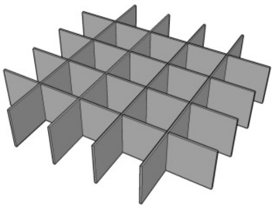

Fig. 1 Deployable cardboard grid.

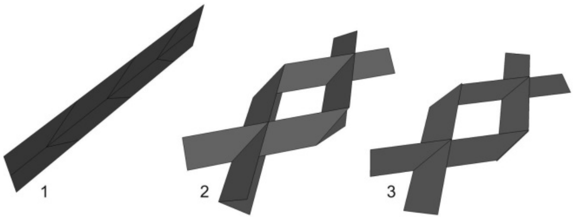

Fig. 2 Deployment sequence of the Tachi-Miura polyhedron.

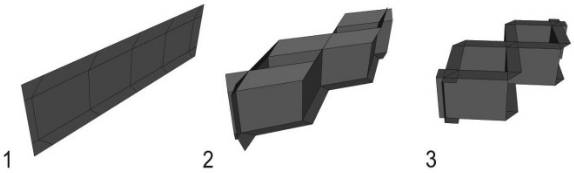

Fig. 3 Deployment sequence of a strip of the grid structure.

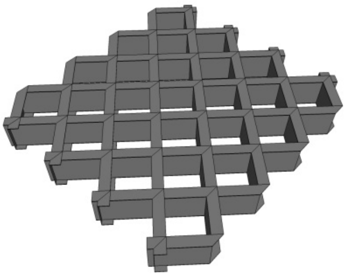

Fig. 4 Deployable grid structure.

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Computational Problems Related to Paper Crane in the Edo Period

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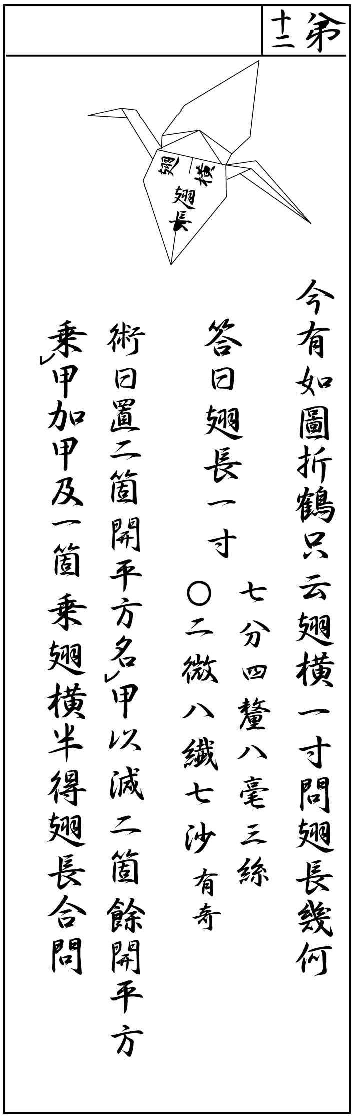
Keywords: Wasan(Japanese mathematics), History of origami, Paper crane (Origami crane)

Wasan (Japanese Mathematics) is a kind of mathematics that was developed uniquely in Japan from the seventeenth to nineteenth century.
Despite its shortcomings in systematization and logical proofs, its achievements are comparable to the contemporary Western counterpart that Wasan was in virtually no communication with. One notable fact about Wasan is that it had spread among all classes in the feudal regime. It was taught in private schools all over Japan as not only practical calculations or measurements but also sophisticated problems purely for fun. The fact is shown in the custom called Sangaku where they offered to a shrine or a temple a board on which they colorfully described geometric or other mathematical problems along with their solutions.

Among such Sangaku and some books at that time, we can find a few problems related to origami, such as problems about figures folded from rectangles or triangles, calculations of the centroids of such figures, calculations about the gravity centers of folded planes, and calculations about the origami crane or other models.
I will look into calculation problems about the paper crane and discuss the following issues,
(1) The relationships to the calculation of Pi, which was one of the most important problems in Wasan.
(2) The relationships between the diffusion of such problems and the sectionalism of Wasan.

Wasan was developed in the same era as origami was. Through this study, I would like to shine a light on an aspect of origami as a culture in relation to Wasan, at the same time on characteristics of Wasan in relation to origami.



弟十三

今有如圖折鶴只云翅横一寸問翅長幾何

答曰翅長一寸（七分四釐八毫三絲〇二微八纖七沙有奇）

術曰置二箇開平方名レ甲以減二箇餘開平方

乗レ甲加甲及一箇乗翅横半得翅長合問

A traced drawing of
“Sanpo Mi no Kagen”
(WATANABE Kazu,1830)

Question : Calculate the length of the wing when its width is 1.
Answer : 1.74830287 and fractions
Procedure : $\frac{\sqrt{2-\sqrt{2}}\cdot\sqrt{2}+\sqrt{2}+1}{2}$

“Sangaku” at Ichinomiya Shrine in Chiba prefecture (1877)

A survey and recent results about common developments of two or more boxes

Ryuhei Uehara[*]

The author with colleagues has investigated common developments that can fold into several incongruent orthogonal boxes. It was shown that there are infinitely many orthogonal polygons that fold into two incongruent orthogonal boxes in 2008 [2]. In 2011, all common developments that can fold into two boxes of size $1\times1\times5$ and $1\times2\times3$ are listed and its number is 2263 [1]. Among them, only one orthogonal polygon can fold into three boxes of size $1 \times 1 \times 5$, $1 \times 2 \times 3$, and $0 \times 1 \times 11$ (Figure 1). It remained open whether there exists an orthogonal polygon that folds into three boxes of positive volume. In 2012, an affirmative answer to this open problem is given (Figure 2[1]) [3]. The construction idea can be generalized, and hence there exists an infinite number of orthogonal polygons that fold into three incongruent orthogonal boxes. So far, the smallest common development of three boxes has area of 532 unit squares. Now we are going to find much smaller and simpler developments that fold into three boxes by admitting non-orthogonal folding. More precisely, we are now going to list all common developments that can fold into $1 \times 1 \times 7$ and $1 \times 3 \times 3$. The surface area is 30, and which may admit to fold into another box of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$. In fact, interestingly, there are some polygons that fold into $1 \times 1 \times 7$ and $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$. We will give positive or negative results about this area 30. That is, we will give if there is a polygon that folds to a box of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ among all common developments of $1 \times 1 \times 7$ and $1 \times 3 \times 3$. We are running a program that solves this problem on a supercomputer, and we will give the answer on the conference.

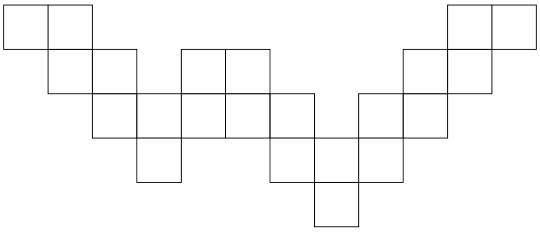


Figure 1: Polygon that folds into three boxes of size $1 \times 1 \times 5$, $1 \times 2 \times 3$, and $0 \times 1 \times 11$

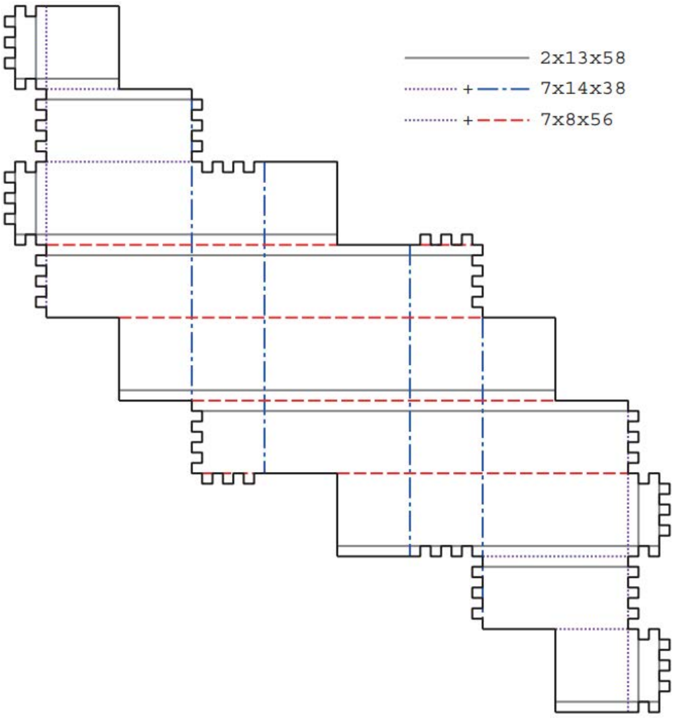


Figure 2: Polygon that folds into three boxes of size $2 \times 13 \times 58$, $7 \times 14 \times 38$, and $7 \times 8 \times 56$

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[*] School of Information Science, JAIST, Japan, `uehara@jaist.ac.jp`

[1] Available at `http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf`

Spiral fold from a tape

Tomoko Fuse
Origami Artist, Independent

When you fold a tape once appropriately, marks（ex.P-1,P-2 and P-3) appear to fold for a next step (Fig.1). As you folded repeatedly in the same way, it is possible to fold helices (Fig.2). There are two folding patterns called “folding once” and “folding twice”. Here, I describe the folding twice; we can produce helices by repeating a set of two types of folds (I will presnt both folding pattern in the conference). At first, we need to fold a paper toward to one of the marks as a target point. Second, we fold in the way that the folding angles of two folds are the same. By repeating these folding stepes, we can finally produce a spiral with regular polygons. We can fold a regular tetragon, regular pentagon, regular hexagon and regular octagon continuously by changing the marks（Fig.3). Various helices can be produced by using this folding technique without designing specific folding angels and lines. That is a wonderful point of Origami. We can fold the helices by hands without protractors, scales, computers and etc.

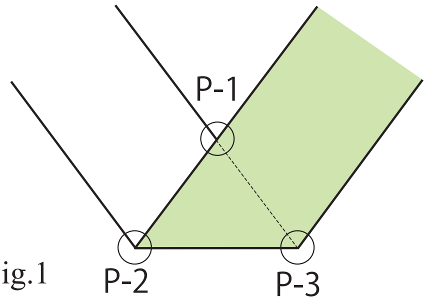

Fig.1

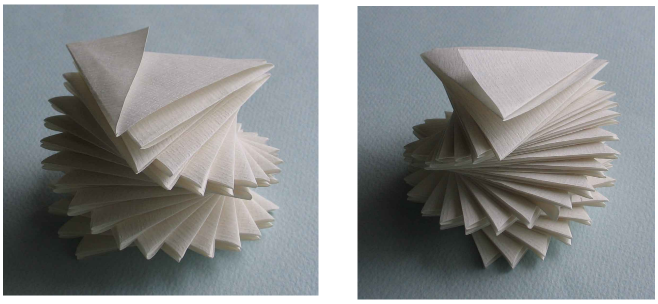

Fig.2

Fig.3

DESIGN OF DOUBLE-WALLED ORIGAMI-CORE AIRCRAFT FUSELAGE SHELL

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Weight reduction is a constantly-pursued goal by aircraft designers. Although double-walled core structures, such as honeycomb sandwich structures, possess superior specific strength, they have only been used as secondary components in aircraft and have not been used in the aircraft fuselage shell. The most important reason for it is that honeycomb structures suffer from an undesirable moisture accumulation problem whereby the condensed moisture is trapped inside the sealed hexagon cells leading to deterioration of the mechanical performance over time. Double-walled structures with origami cores do not have the moisture accumulation problem due to the existence of open channels in such structures, and as a result have been proposed to use as a promising sandwich fuselage concept named VeSCo by Airbus.

As a preliminary study, this paper focuses on the geometric design of origami cores for aircraft fuselage usage. In particular, we developed a method, known as the vertex method, which enables one to readily construct origami of various fold patterns and geometries [1]. Based on the method, a design protocol of origami cores for double-walled aircraft fuselage shell is proposed, which includes (1) prescribing the input forms in the x-z and y-z planes of a Cartesian system, (2) solving a non-linear equation system to obtain the design parameters according to the fuselage geometries, and (3) using the Vertex Method to acquire the final origami core design.

To illustrate the design procedure, we take as an example a simplified fuselage segment with a circular cross-section whose outer and inner radii are R_{out} and R_{in}, respectively, external and internal wall thickness are t_{ex} and t_{in}, respectively, and length is l_{seg}, as shown in Fig. 1.

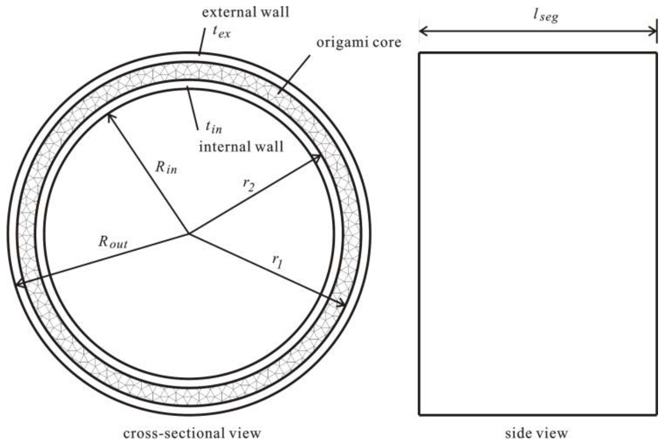

Fig. 1 A simplified circular fuselage segment

The outer and inner radii of the origami core are found by

$$r_1 = R_{out} - t_{ex} \tag{1}$$

$$r_2 = R_{in} + t_{in} \tag{2}$$

respectively. The *x-z* plane inputs for the vertex method are given by

$$\boldsymbol{V_i^x} = [\textstyle\sum_{k=1}^{i}\left(\frac{1-(-1)^k}{2}a + \frac{1+(-1)^k}{2}b\right), 0, -h + \textstyle\sum_{k=1}^{i}(-1)^{\frac{i-1}{2}}(1-(-1)^i)h]^{\mathrm{T}}, i = 1, \ldots, m \tag{3}$$

where *a* and *b* satisfy

$$\sum_{k=1}^{m}\left(\frac{1-(-1)^k}{2}a + \frac{1+(-1)^k}{2}b\right) = l_{seg} + a \tag{4}$$

Solve for r and δ the following equation set.

$$r_2^2 = (r-\delta)^2 + h^2 + h^2\left(\frac{(r+\delta)\,sin\frac{\pi}{N}}{(r+\delta)\,cos\frac{\pi}{N} - (r-\delta)}\right)^2 \tag{5}$$

$$r_1^2 = (r+\delta)^2 + h^2 + h^2\left(\frac{(r-\delta)\sin\frac{\pi}{N}}{(r+\delta) - (r-\delta)\cos\frac{\pi}{N}}\right)^2 \tag{6}$$

where *N* is an arbitrary integer larger than 2. Then, the *y-z* plane inputs for the vertex method are given by

$$\boldsymbol{V_j^y} = [r + (-1)^j\delta]\begin{bmatrix}0\\ \sin\left(\frac{j\pi}{N}\right)\\ \cos\left(\frac{j\pi}{N}\right)\end{bmatrix}, j = 0,1,\ldots,2N+2 \tag{7}$$

Finally, Eqs. (3) and (7) are taken as the inputs for the vertex method to generate the final origami core design. The resulting origami core is shown in Fig. 2(a), where R_{in}, R_{out}, t_{in}, t_{ex}, l_{seg}, m, h, a and *N* are taken as 18, 22, 1, 1, 11, 12, 1, 1 and 30, respectively. Fig. 2(b) shows the project of the vertices in *y-z* plane. Note that all vertices locate on two circles with radii equal to 20 (i.e. r_1) and 19 (i.e. r_2), respectively.

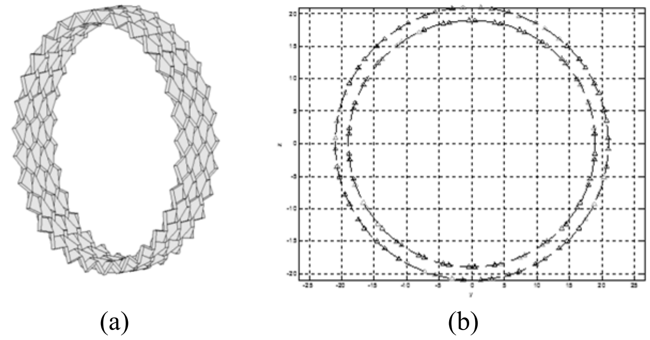
(a) (b)

Fig. 2 (a) The resulting origami core. (b) Projection of the vertices in *y-z* plane.

By varying the *x-z* and *y-z* plane inputs, various origami cores (including those with curved fold lines) that fit into the space between the external and internal walls of fuselages with circular or non-circular cross sections can be designed. The work presented in this paper provides aircraft engineers a powerful tool to gain quick access to a large pool of origami cores for a given fuselage which will serve as the basis for the subsequent structural analysis and optimization.

Reference

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Composing Right Frusta to fold Axially Symmetric Origami

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Abstract

An original method has been derived to use origami to fold three-dimensional polyhedra that have axial symmetry. In origami terms, the polyhedron is folded by sequentially grafting right frusta on top of each other. That is, a frustum is extruded and then its surrounding paper is used to fold the next frustum, until the entire polyhedron has been extruded.

The crease pattern of the polyhedron is generated by *composing* together the crease patterns of the individual frusta. An algorithm has been devised to compute the crease pattern that results from grafting two frusta together, basically by "splitting apart" the second crease pattern using the pleats within the first crease pattern. This algorithm is recursively applied to compute the crease pattern of the polyhedron. A computer program has been written, allowing users to specify a target solid and generating its crease pattern.

The sequential grafting modularizes the design and folding processes, as compared to existing *ad hoc* design algorithms. The modularization allows more versatile design from combining individual frusta in novel ways. The process of folding from the crease pattern is also sequential and thus easier, as opposed to a single-step crease pattern collapse.

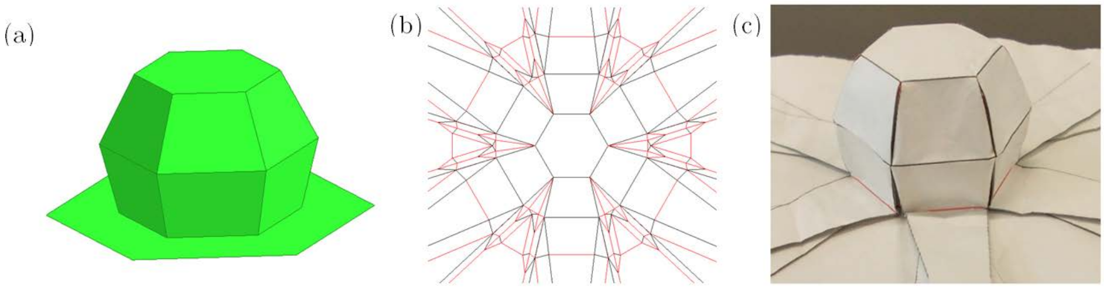

Figure 1: Folding an axially symmetric polyhedron.
(a) Target solid (b) Crease pattern (c) Folded product

Using Origami to Enrich Mathematical Understanding of Fractals

Ali Bahmani* Kiumars Sharif Andrew Hudson
University of Tehran *University of Tehran* *University of California, Davis*

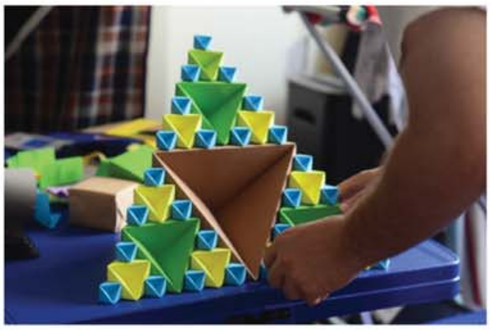

Figure 1: A modular Sierpinski fractal

In this paper we present some ideas about using origami to teach students about fractals and related concepts. Our research focuses on students in the 7th-10th grades at the National Organization for Development of Exceptional Talents (NODET) program schools in Iran. NODET's supplementary math curriculum employs a variety of educational approaches, but we have chosen to focus on a project-based learning model. After some introduction to a topic is given, students are assigned a few activities by the teacher, and then choose a larger creative project with the guidance of the teacher to pursue on their own.

After taking an informal survey of math teachers in the NODET program, we found that these students often struggled with Infinity, Limits, and related concepts. Origami is well-suited to demonstrating examples of convergent sequences and series, and provides a hands-on approach that we think will help some of these students. We will present some preliminary conclusions about the success of this curriculum based on our own observations, as well as student feedback. We will also present our lesson plan, discuss how we designed it, and show some examples of student work.

These lessons occurred as part of a Origami+Math series taught at the NODET schools. We presented three activities: making a popup staircase, folding spirals from a strip of paper, and then working together to fold a modular Sierpinski Fractal. Each of these activities was used to discuss some of the mathematics behind the geometrical phenomena being presented. We also discussed how these concepts show up in Iran's rich geometrical art heritage. Students who wished to pursue origami further were then exposed to some recursive folding techniques.

Figure 2: (Left) Interior of Sheikh Lutfollah Mosque (Right) Examples of recursive models: (Top) Shuzo Fujimoto's Hydrangea and (Bottom) Chris K. Palmer's Flower Tower

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Counting Miura-ori Phantom Foldings

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Western New England University

A crease pattern $C = (V, E)$ (viewed as a plane graph with vertex set V and creases (edges) E) together with a mountain-valley assignment $\mu : E \to \{M, V\}$ is called a *phantom fold* if every vertex in V is flat-foldable using μ. In other words, a phantom fold is locally flat-foldable but globally might cause the paper to self-intersect.

We consider the problem of enumerating the different phantom folds of the classic Miura-ori crease pattern (Figure 1). Specifically, we aim to count the number $M(m, n)$ of ways to assign mountains and valleys to the creases so that each vertex in a m-parallelogram by n-parallelogram Miura map fold will be able to locally fold flat. Recurrence relations and closed formulas are found for small m and arbitrary n, and we will see why finding a general formula for $M(m, n)$ seems intractable. However, the array of numbers generated by $M(m, n)$ is equivalent to the number of ways to properly 3-vertex-color a m-vertex×n-vertex grid graph with one vertex pre-colored (Figure 2). Statistical mechanics then tell us that as the number of parallelograms N gets very large (say, around 10^{23}) then $M(m, n)$ will be approximately $(4/3)^{3N/2}$.

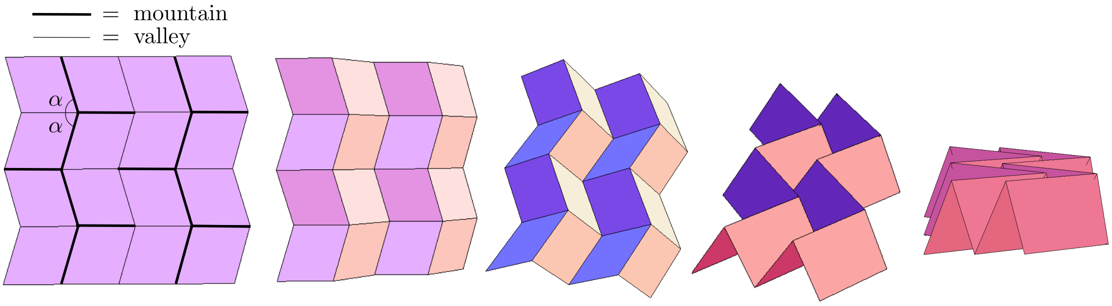

Figure 1: A 4 parallelogram × 4 parallelogram Miura-ori with the standard MV assignment.

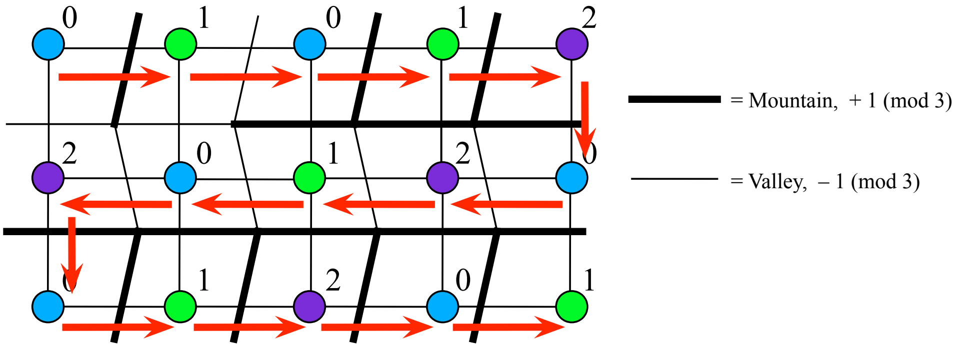

Figure 2: An example of the correspondence between proper 3-vertex colorings of a 3 × 5 grid graph and a 3 × 5 Miura-ori.

FOLDING AUGMENTED
A conceptual design method to integrate structural folding in architecture

Pierluigi D’Acunto, Juan José Castellon Gonzalez

Origami, the art of folding paper, intuitively shows how folding could be employed to generate continuous but differentiated geometries that have also the potential to perform structurally. At the scale of architecture, this is translated into the possibility of employing folding to generate forms that are able to address diverse spatial and programmatic necessities and, at the same time, to produce efficient structures capable to resist the external applied loads by form. Thanks to its inherent properties, Origami has been widely used in architectural education as a source for proto-architectural design, usually relying on design methods that involve the construction and manipulation of physical models based on sheet-materials folding [1]. In these design explorations, however, the question of structure and space can be addressed in an intuitive way only. The goal of this presentation is to introduce a novel conceptual design method to consistently integrate structural folding in architecture. In particular, the proposed method is aimed at supporting the design process starting from the early conceptual stages. In this framework, folding is regarded as a way to effectively combine architectural and engineering thinking towards a holistic approach to design.

The proposed conceptual design method is based on a three-dimensional design process grounded on simple geometrical operations. Unlike other approaches to digital folding based on the simulation of rigid Origami [2], here a surface is virtually folded within a predefined tetrahedral grid, thus generating the envelope of an enclosed architectural volume that at the same time performs as a structure. Specifically, the process includes four main operations. These ones should not be necessarily intended as implemented in a fixed order. In fact, it is possible to execute the operations in a non-sequential way, while keeping the overall consistency of the design process:

A. Operation A consists in the generation of a tetrahedral three-dimensional grid: in this phase, the topology of the architectural space is defined by choosing how many tetrahedra are included in the grid and how they are connected to each other, following specific spatial and programmatic requirements as design inputs; the employment of a tetrahedral grid is here of crucial importance to attain the global structural stability of the folded surface that will be generated with the following operation; indeed, a tetrahedral grid has in 3D an analogous property of undeformability that a triangular grid would have in 2D.
B. With operation B, a continuous surface is virtually folded within the previously defined tetrahedral grid; that is, a surface that connects adjacent faces of the tetrahedral grid is here designed; through this operation it is possible to generate the envelope of an enclosed architectural volume. Being produced out of an individual folded surface, the resulting architectural space is continuous.
C. With operation C, by moving the vertices of the tetrahedral grid, the shape and dimensions of the folded geometry can be adjusted to meet specific programmatic and structural requirements or to adapt to certain boundary constraints. This operation introduces differentiation into the process.
D. Operation D allows for the structural logic of the folded surface to be revealed and, with it, the relationship between the flow of forces within the system and its overall form; by observing that in folded structures the main internal forces tend to travel along the folded edges [3], a load-bearing mechanism can be activated inside the folded surface, which is equivalent to a pin-jointed truss framework where the folded edges represent the elements of the truss, loaded either in tension or compression [4]. With this operation, the integration between structure and architecture is attained and the internal force flow within the folded structure can be evaluated by means of graphic statics.

Being based on geometrical operations only, the proposed design method for structural folding in architecture is material and scale independent. This allows for the same strategy to be applied to various design scenarios.

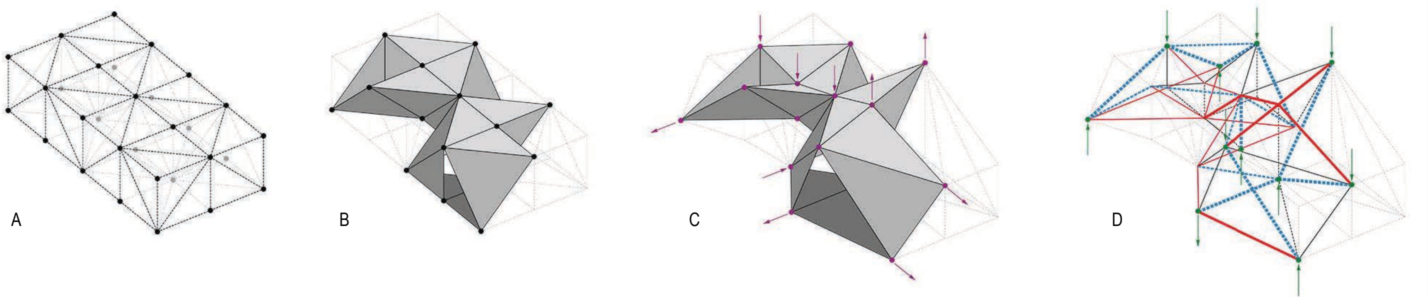

Fig. 1 – Conceptual design method for structural folding in architecture: A. Definition of the tetrahedral grid; B. Generation of the folded surface; C. Adjustment of the geometry; D. Evaluation of the internal force flow by means of graphic statics (blue-compression, red-tension)

In order to make the proposed conceptual design method operative, a parametric digital tool is under development, which gives the designer the opportunity to take full advantage of the interplay between spatial and structural potentials of folding in a synthetic and easy way. That is, by working within a three-dimensional software environment such as McNeel Rhinoceros 3D and Grasshopper, the designer is able to interactively modify the geometry of the folded surface based on a spatial grid, while having a real-time direct feedback on the distribution of the tension and compression forces within the structure.

Keywords:
Folding, Conceptual Design Method, Parametric Digital Tool

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Rigid Flattening of Polyhedra with Slits

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Thomas Hull[‡] Anna Lubiw[§] Tomohiro Tachi[¶]

In many real-life situations we want polyhedra or polyhedral surfaces to flatten—think of paper bags, cardboard boxes, and foldable furniture. Although paper is flexible and can bend and curve, materials such as cardboard, metal, and plastic are not. The appropriate model for such non-flexible surfaces is "rigid origami" where the polyhedral faces are rigid, and folding occurs only along pre-defined creases. In rigid origami, flattening is not always possible, and in fact, often no movement is possible at all. In particular, Cauchy's theorem of 1813 says that if a convex polyhedron is made with rigid faces hinged at the edges then no movement is possible. Connelly showed that this is true even if finitely many extra creases are added.

However, cutting the surface of the polyhedron destroys rigidity and may even allow the polyhedron to be flattened. We initiate the study of "rigid flattening" of a polyhedron: continuous flattening with rigid faces after the addition of finitely many cuts and creases. We require that the final flat folding be a flat folding of the original polyhedron, i.e. that every cut closes up at the end of the flattening process.

It is an open question whether every convex polyhedron has a rigid flattening using cuts that do not disconnect the surface. More generally, we might ask to minimize the length of the cuts. We show that a regular tetrahedron with side lengths 1 can be continuously flattened with rigid faces after cutting a surprisingly small slit of length .046 and adding a few extra creases. The flattening process has only one degree of freedom. We use Mathematica to model the motion and verify that no self-intersections occur.

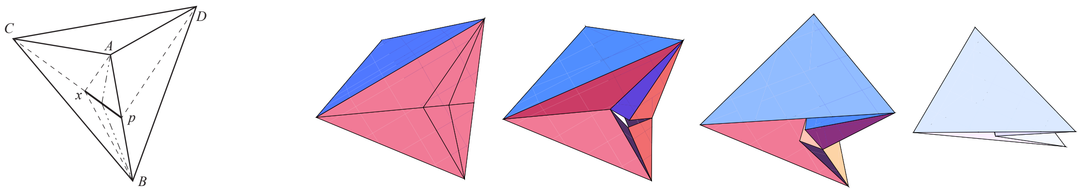

Figure 1: Rigid flattening of a regular tetrahedron after cutting a slit of length $\frac{1}{2\sqrt{3}}$ to the incenter x.

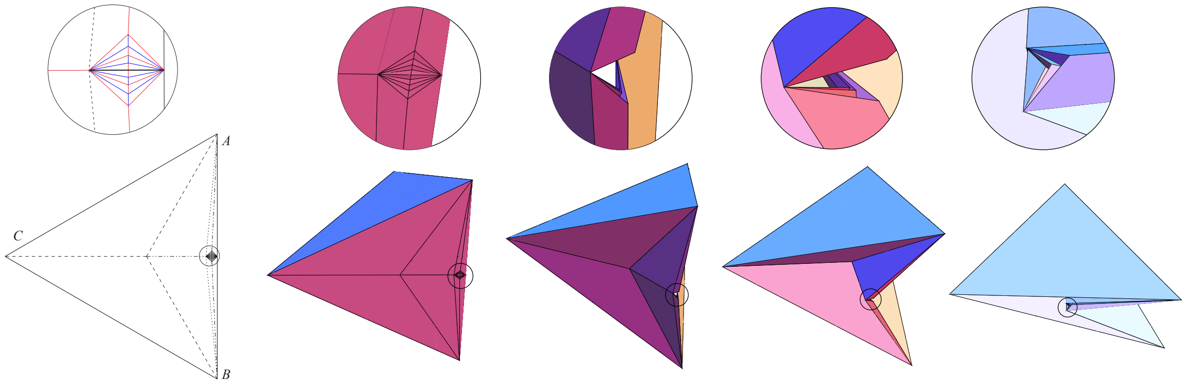

Figure 2: Rigid flattening of a regular tetrahedron after cutting a slit of length .046. The top row shows close-ups of the bottom row. On the left are the slit and crease pattern on face ABC; in the close-up, red lines are mountain folds, blue lines are valleys, and the dashed creases switch between the two.

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The Mathematical Legacy of Humiaki Huzita

Emma Frigerio (*)

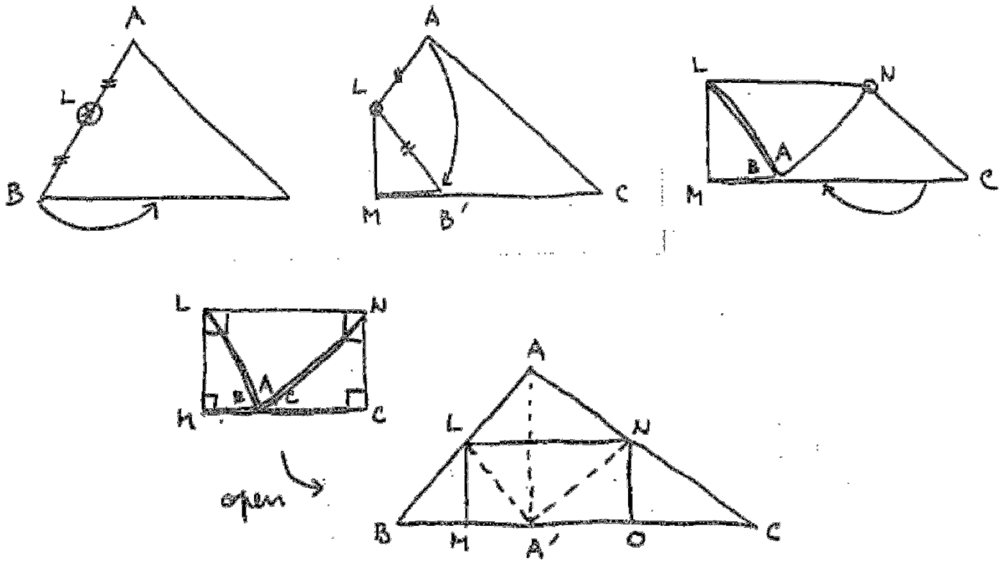
Humiaki Huzita (1924 - 2005) was a pioneer in the field of the mathematics of origami and a bridge builder between Japanese and Western folders and scholars. A physicist at the Italian Institute of Nuclear Physics in Padua, he had a profound understanding of Euclidean geometry and a strong curiosity about its relationship with paper-folding, and made the work by masters such as Abe and Fujimoto known to the non-Japanese-speaking world.

He is best-known for his six axioms of the geometry of flat folding, which he developed together with Scimemi (a mathematician at the University of Padua): they adapted the Galois-theory proof on ruler-and-compass construction to the more general setting of origami constructions. Later, Hatori added a seventh axiom, which indeed describes a folding procedure sometimes used in origami, but does not extend the set of possible geometric construction; as a matter of fact, Hatori's axiom can be performed with ruler and compass. What makes origami geometry more powerful than ruler-and compass geometry is Huzita's sixth axiom, of which all others can be considered special cases (in other words, the six –or seven– axioms are NOT minimal).

Huzita's contribution to the mathematics of origami goes beyond its axioms: in his work, he sought to construct a sound foundation for origami geometry. For instance, he proposed a definition of parallel lines (and are parallel if there is a line perpendicular to both and) and a substitute for Euclid's Fifth Postulate (if a quadrilateral has three right angles, then the fourth angle is right as well); he then showed how it can be used in the proof of other theorems. Moreover, he envisioned further developments such as multiple folding, thus anticipating, in some sense, Lang's angle quintisection.

Last but not least, in 1989 he organized the first of these meetings in Ferrara, where it all began: as a matter of fact, Margherita Piazzolla Beloch, whose earlier work on the subject had gone almost unnoticed, was a professor at the University of that town in the Thirties. Huzita went on to publish himself the Proceedings, where he reprinted earlier works by her and others as well.

I had the privilege to discuss mathematical issues with him a number of times, which greatly contributed to my interest in the mathematics of origami, and I always appreciated his ability to look at things from novel points of view. In this paper, I give his work a retrospective look, almost ten years after his death, and attempt to pay him the tribute he deserves.



Huzita's figures for proving that, in any triangle, its angles sum to two right angles.

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Origamic Architecture and Van Hiele Model in the Graduate Classroom of Descriptive Geometry

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Abstract

The objective of this article is to present an alternative methodology to teach geometry concepts using *Origamic Architecture* (OA) as a motivational strategy. During the years as a geometry professor, the author observed that her graduate students have experienced increasing difficulties in spatial orientation and visualization. This situation is consequence of lack of basic geometry lessons during primary and secondary school programs in Brazil during the last thirty years.

However, the Brazilian government has started to change this situation and geometry concepts and constructions will return to school. So the mathematics teachers should be able to understand geometric structures, to produce them and to have knowledge of educational strategies to teach them.

The presentation of geometric concepts and the development of spatial visualization and representation are some of the challenges of descriptive geometry disciplines. Mental and graphic abilities should be combined in order to stimulate the student to observe and to represent the forms in an organized way. The use of OA in geometry disciplines may be a motivational factor in order to develop a stimulating learning process. In this way, it is possible to discover the beauty of mathematics by understanding and producing structures through geometry and *Origamic Architecture*.

Therefore, this case study presents the methodology applied during formation of primary and secondary mathematics teachers and also presents the results of an investigation using OA in order to develop spatial perception through a rational and a sensitive method. The strategy adopted was the Van Hiele Model of Development, by observing and considering the five phases of learning, which are:

1) Interrogation: teacher asks questions in order that students explore the topics;
2) Orientation: exploration of the topics by the students;
3) Explanation: students and teacher express their views and comments together by listing the basic comprehension;
4) Free orientation: students explore other possibilities and resolve more complex tasks;
5) Integration: a summary is made about what they have learned with the activity.

During this process, the role of the teacher is to provide orientations and help in the experiences of the students, but the main work is made by them, improving the process of learning. The strategy adopted is the presentation of some basic models during the first phase, followed by the orientations and explanations observed. Afterward, the complex task required by the teacher, is presented and some results that illustrate the efficiency of this methodology as well.

Concerning Origamic Architecture as a particular activity, this article reports some observations about the specific concepts and skills developed through this technique, the students behavior observed while this approach, and the difficulties presented during this study.

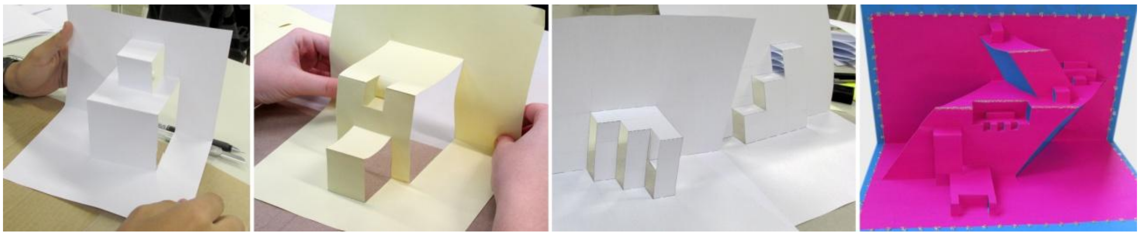

Fig. 1: Topics explored by the students (from simple to variations) and one of the results about the complex task.

Keywords: Origamic Architecture, geometry learning process, Van Hiele Model, pop-up structure, spatial perception.

Geometric Strategies to Create Origamic Architecture Models

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Abstract

The Origami Creases and The Kirigami Cuts, together in an oriental technique called *Origamic Architecture* (OA), has intrinsic relationships with geometry. This new concept was introduced by Masahiro Chatani [1] and it is also known as 3D Kirigami or Pop-up Structure.

There are a number of important studies about software developments and computational language to help OA design. However, geometric constructions and design concepts for graphic designers (students and professionals) are very important topics to be studied and practiced during creation of new models using drawing skills and tools.

This present study concerns about a miscellany of some recent works about OA geometry [2] [3] [4] during the process of creating new designs, especially when the objective is the development of spatial perception of the figure and correct position of cutting lines and creases. Some initial guidelines were established for design of OA through visual elements such as: point, line and shape in two-dimensional (2D) plane in order to obtain the originally planned three-dimensional (3D) model, concentrating on 90^{o} open type models. Principles of form and design [5] are also applied during the creation process.

Some models are presented to illustrate the constructive parameters detected as well as some models elaborated by the author and by graphic design students, who have never made any OA model, showing the viability of the guidelines established.

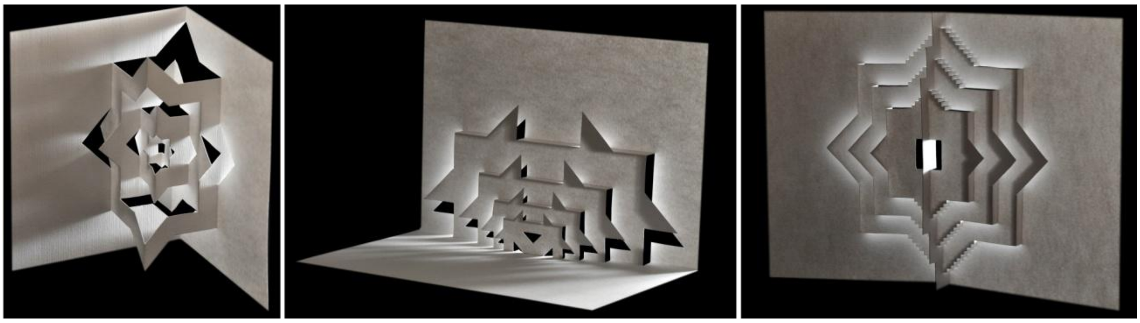

Fig. 1: Some models created by the author.

Keywords: Origamic Architecture, constructive guidelines, geometry, graphic design

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Self-Folding of Reconfigurable Complex Structures using Programmable Active Laminates

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Engineering inspired by origami, such as the practice of creating useful three-dimensional structures through folding and fold-like operations applied to initially two-dimensional entities, has the potential to impact several areas of design and manufacturing. In some situations, however, it may be unfeasible to apply external manipulations to produce the desired folds (e.g., as in remote applications such as space systems). In such situations, self-folding capabilities are necessary. In this work, we utilize origami principles to generate single planar surfaces that can be folded into complex structures along arbitrary (i.e., no hinges or pre-engineered locations of folding) fold lines. The surfaces considered herein are composed of shape memory alloy-based laminated composites. Shape memory alloys (SMAs) are materials that can change their shape by thermal stimulus and such shape changes are reversible. The aforementioned characteristics of SMAs allow us to utilize such materials to construct reconfigurable complex structures that can be self-folded without the need of external forces. The generation of surfaces that can be folded into the desired structures is done utilizing origami design software such as Tachi's Freeform Origami[1]. Also, we have developed an in-house fold pattern design software that is capable of generating both straight and curved fold patterns. After designing fold patterns, we convert them into a finite element mesh that can be analyzed in finite element analysis (FEA) software. Then, we utilize physics-based FEA to determine whether by appropriately heating the planar unfolded surface it is possible to fold it into the desired complex structures. The results show that the self-folding structures can be raised against gravity, folded, and reconfigured via thermal stimulus. We demonstrate this by folding towards multiple designs generated from Freeform Origami and our in-house origami design software.

Figure 1 shows the folding capabilities of the proposed self-folding active laminate. The laminate is folded into a Resch pattern generated by Freeform Origami.

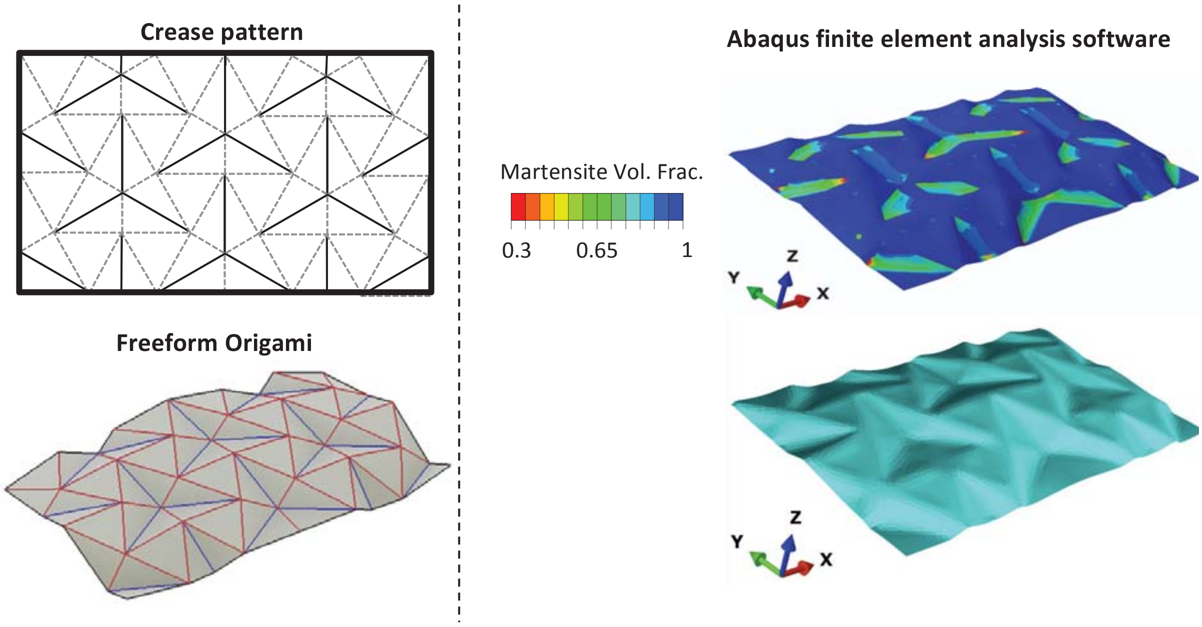

Figure 1. Demonstration of self-folding programmable sheet folding into a Resch pattern foldcore generated by Freeform Origami. The contour plot represents the volume fraction of SMA that has been transformed from martensite to austenite due to thermal stimulus.

Reference:

1. Freeform Origami by Tomohiro Tachi, http://www.tsg.ne.jp/TT/software/

Folding Self-similar Tilings Based on Prototiles Constructed from Segments of Regular Polygons

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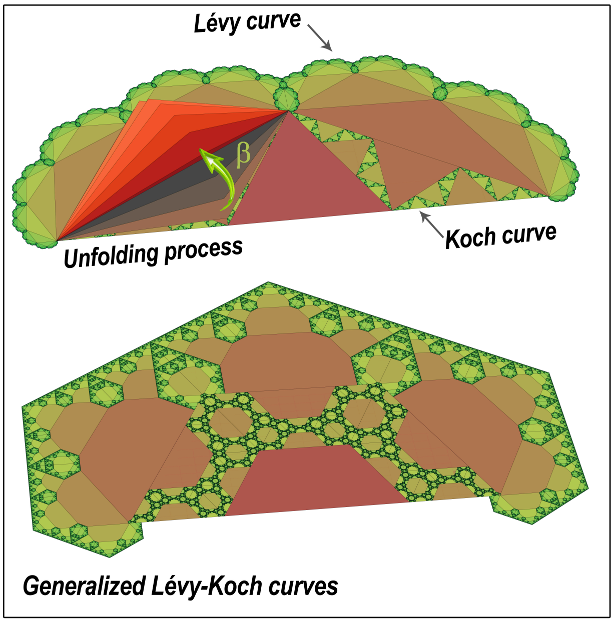
In 1938, Lévy published a work entitled “Plane or Space Curves and Surfaces Consisting of Parts Similar to the Whole”. It contained several hand drawings of plane fractal curves, now called Lévy curves, generated by unfolding outwards the smaller triangular components of the Koch curves. As he noted, these planar fractal curves were particular cases of a family of three-dimensional Koch-like curves obtained from unfolding the triangular components by a certain angle at each generation. In this paper we generalize this family of curves using n-gons instead of just using the triangular components of the Koch curve. We provide the equations for the scaling-factor as function of the N-fold symmetry and the type of n-gon. We illustrate the folding process of several self-similar tilings with N-Fold symmetry. And we reconsider the study of these objects as paper-folded fractal origamis.



The scaling factor of the prototiles is given by

$$s = \sin\left(\frac{\pi}{f}\right)\csc\left(\frac{\pi\, n}{f}\right)$$

where n stands from the $n-$gon type
and f for the $N-$fold symmetry

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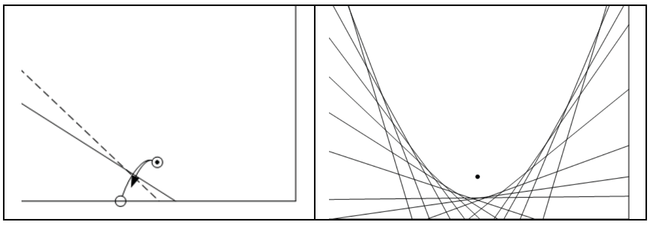
Origami for connecting mathematical ideas and building relational understanding of mathematics

Abstract

At 5OSME, together with Lam, I explored how origami can be used as convenient and accessible apparatus for learning and applying mathematics. We discussed some common difficulties that learners encounter when working on mathematics through origami. We also recommended teaching strategies with origami that can challenge misconceptions and nurture conceptual understanding, leading to deep mathematical learning.

Research suggests that highly effective teachers make connections within mathematics (Askew et al. 1997) that support the development of relational understanding (Skemp 1976). In this paper I exemplify how a simple and accessible origami starting point can be used to make connections in mathematics and develop relational understanding. I draw on experiences of folding the envelope for a parabola with in-service teachers, pre-service teachers and learners aged 14 and upwards. Whilst students with a traditional Euclidean geometry background might be content to accept the folding procedure as one that must generate a parabola, for most modern students their first experience of a parabola is as the shape of the graph of a quadratic function. I describe how learners move from the physical experience to the abstract mathematical model that convinces the modern mathematician that the envelope is that of a parabola.



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ACTIVATING LEFT AND RIGHT BRAIN FUNCTIONS WITH ORIGAMI

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ABSTRACT

Being involved into the art of origami for many years, we have noticed, that it influences our mental condition as well as motor ability of both hands. As psychologists, we have decided investigate what influence origami can render on psychological and psychophysiological characteristics of people intensively involved into paper folding.

The process of folding paper involves both hands without condition of dominating any of them and without necessity of holding tools; such activity can be referred to as bimanual asymmetrical activity. Such coordinated work of hands under visual control requires sufficient activity of the brain and harmonious work of its various structures.

Origami as asymmetrical bimanual activity and visual-spatial activity with high emotional and productive orientation represents a good basis of a psychomotor training to increase interaction of left and right hemispheres of the brain and to develop motor, intellectual and creative abilities.

We conducted this Ph.D. thesis research on basis of faculty of psychology of Rostov-on-Don State University, Russia. 137 pupils (67 boys and 70 girls) of comprehensive schools were taking an intensive origami course during a week vacation in the absence of other educational subjects (25 hours: everyday 5 educational hours for 5 days). None of them had origami skills prior to the beginning of the research. Psycho-diagnostic tests were conducted in 3 stages: before, in the middle and after this intensive origami training.

According to the results of this research, for the first time was experimentally proved that the origami training creates conditions of intensive interaction of brain hemispheres and allows effectively develop motor abilities of both hands, intellectual and creative abilities at children 7-11 years old.

key phrases:

left and right brain
origami as intensive bimanual training
intellectual and creative abilities

Screw Algebra Based Kinematic and Static Modeling Approach for Origami Enabled Structures

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Origami has been a long-standing and unique sculptural art varying from simple folds to the more complicated modern origami. The intricate origami is produced by folding one regular-shaped sheet of paper with combined creases patterns in a variety of flexible ways. The underlying principles for geometric design of origami led to mathematical modelling for various types of stunning origami structures. The recently emerging development of origami-inspired systems showed that better understanding of knowledge for origami in artistic discipline could benefit the study in the engineering disciplines. The kinematic principles were used in the study of foldable origami structures and pop-up book to bridge the gap between engineering and artistic designs. This interdisciplinary study on both origami and kinematic structure shed bright glow and presented an interesting approach for catalyzing new ideas in engineering designs. One of the most attractive features of origami enabled structures is the possibility to design the kinematics and stiffness of the system to maintain specific directions in which the whole structure can be completely folded, thus providing inherent compliance to the origami enabled system. This paper unravels the intrinsic origami and tessellation principles especially the geometry of creases patterns and the stiffness consequent of origami structure composed of creases and panels. The mathematical models of the kinematics and statics associated with various geometry and topology are developed in terms of screw algebra by aggregating single crease's folding characteristics. In particular, motion of panels and creases of origami enabled structures are investigated and analytical models for predicting internal properties of the structure with different configuration are revealed. This in-depth unravelling of principles and approaches for modelling origami structures resorting to mechanism theory in engineering discipline provides fundamental bases for the development of novel concepts and manufacturing techniques of origami-inspired systems. Particularly, the modelling will enable possibility to shrink complex origami-inspired robotic systems to very small size for practical applications with confined working space.

Keywords: Origami enabled structure, kinematics, stiffness, screw algebra

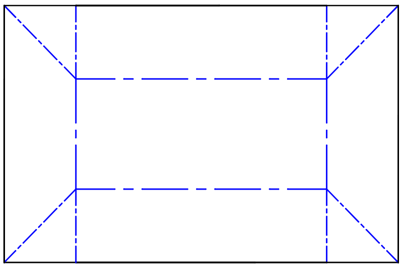

Fig. 1 Standard base of crease pattern in David Huffman's rectangular woven design

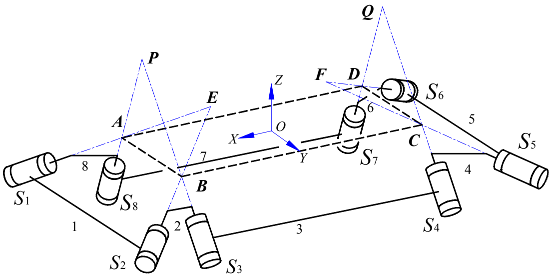

Fig. 2 Modelling of the standard base of creases pattern in terms of Screw Algebra

Base Design of Snowflake Curve Model and its Difficulties

Ushio Ikegami

The author started creating the Koch snowflake curve with theoretical infinite folding around 2000 and presented the 4th iteration of a trial model, with a question to flat foldability of some part of it, at 4OSME[1]. But constructing a crease pattern with an arbitrary iteration order n is still so difficult that its infinite foldability has not yet been established.

The author's recent research found the 4th iteration model flat-foldable, but 5th iteration model's flat foldability remains in question. The crease pattern for trial model of 5th iteration doesn't have enough similarity with 4th one. This suggests that identifying finite generators to construct a crease pattern for any n-th iteration would not get easier even if 5th iteration model were found foldable. Thus we consider that folding the Koch curve requires infinite generators or a large irregular area and finding them through trial and error is unrealistic. Our next focus is the possibility of a simplified Koch curve as infinite folding model with its outline longer than arbitrary length without its size getting infinitesimally small.

In this talk, we will present the basic vertex arrangement design of the model in 3-fold symmetry, crease patterns of 4th and 5th iteration, and some difficulties in folding it infinitely. And regarding a simplified Koch curve model, infinitely foldable two vertices of the curve in particular will also be discussed.

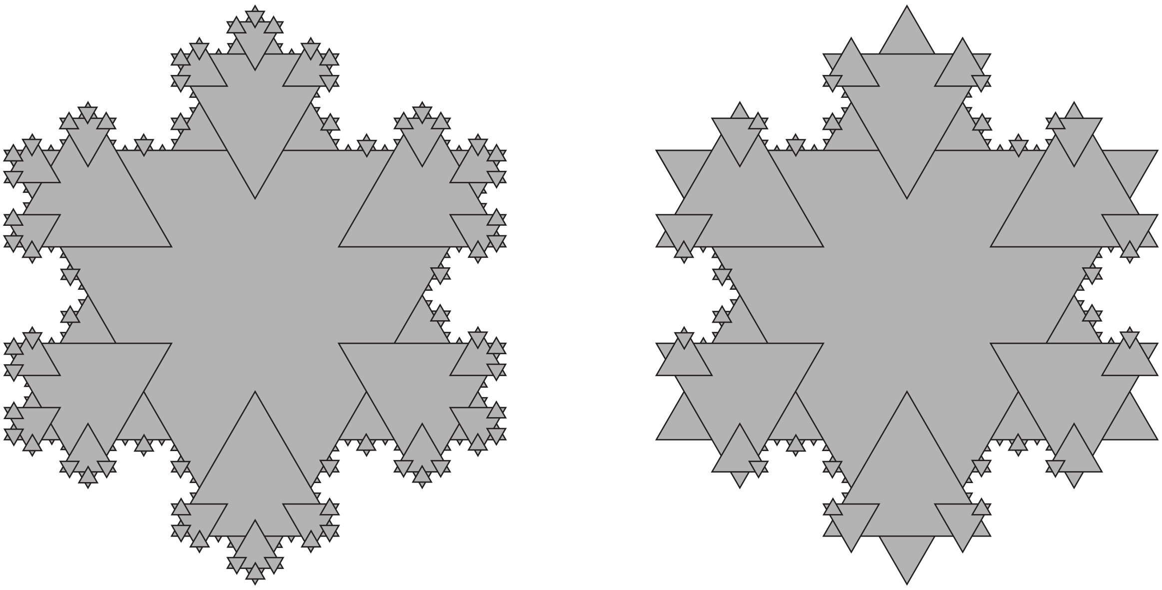

Figure 1
Original design for the snowflake curve model(Left) and its simplified version(Right)

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Modeling Vaults in Origami: a Bridge between Mathematics and Architecture

Caterina Cumino (1), Emma Frigerio (2), Simona Gallina (3), Maria Luisa Spreafico (1), Ursula Zich (3)

How can mathematics help architecture in designing origami vaults?

In this work we consider barrel vaults and their intersections; then simple vaults are cylinders (or cones) of specified cross-section, while compound vaults are formed by two or more simple vaults crossing each other at specified angles. Origami provides a way of modeling a real vaulted structure, thus making its geometry concrete.

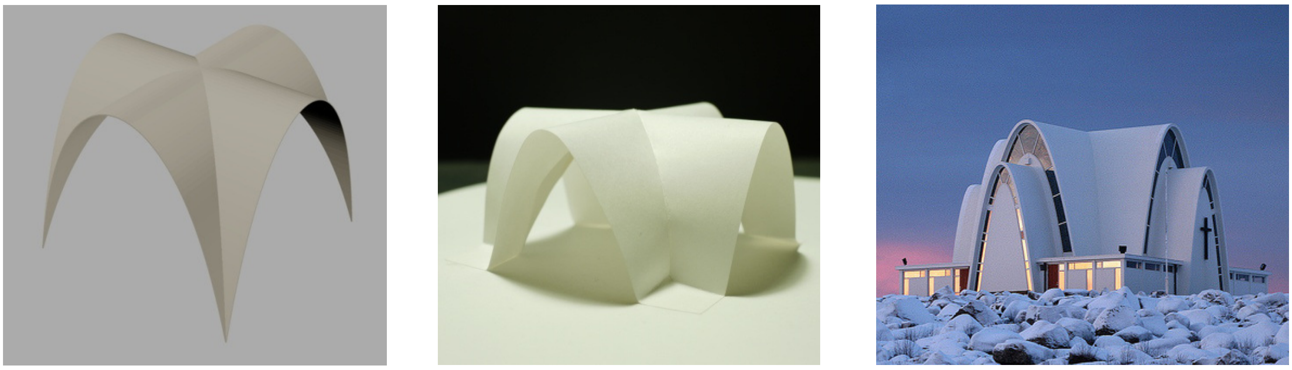
Folding techniques along straight lines and curves allow us to work with ruled surfaces. We restrict our attention to the case of mutually orthogonal cylinders C_1 and C_2 whose intersection curve γ is piecewise planar. More precisely, we assume C_1 as obtained by translation of a curve of equations $z = f_1(y)$, $x = 0$ along the x-axis, with f_1 an even function, so that C_1 is symmetric with respect to the plane $y = 0$; similar assumptions are made on C_2. Once C_1 and C_2 are developed onto a plane, their intersection curve γ generates two curves γ_1 and γ_2, one on each development. Depending on which portions of the cylinders we retain, we get a groin vault or, dually, a cloister vault. Thus, we retain the corresponding portions of the developed cylinders and we arrange them on the plane, in order to obtain the desired crease pattern.

A mathematical computation shows that we can overlap γ_1 and γ_2 via straight folds if, and only if, whenever $x \geq 0$ and $y \geq 0$, the portion of γ in that space region lies in a plane parallel to the plane $x - y = 0$ (dihedral bisector of $x = 0$ and $y = 0$). Then the 3D-model is obtained via curved folds along portions of γ_1 and/or γ_2, together with straight folds. Moreover, if the intersection curve γ is piecewise planar, then, apart from some obvious cases, C_2 is essentially the same as C_1 (as in Philip Chapman Bell's model of a groin vault generated by circular barrel vaults), or a portion of it; in the latter case we obtain barrel vaults with lunettes or with cloister heads. Further, essentially by flipping the cylinders upside-down, concave-down vaults are transformed into a concave-up vaults (for instance, a Western-style cloister vault becomes a pagoda-style vault).

Our study combines various approaches: mathematical description and virtual modeling via AutoCAD and other software (mainly MeshLab and Pepakura Designer) allow us to obtain crease patterns starting from simple vaults with various cross-sections (e.g. semicircles, parabolas, catenaries). When folding a model, ideally the extra paper that does not belong to the vault should lie against it, without occupying the volume defined by it, in view of practical uses; it must be noticed that the position of the extra paper depends on the concavity of the curves involved.

The possible applications of this work are manifold. In the first place, it can contribute to promote a better geometric understanding among students in architectural drawing courses. Moreover, the 2D development of a vault could be used to estimate its area before beginning preservation work, or to produce design objects for cultural merchandising. Finally, a 3D real-size model could be folded onsite from lightweight thin laminates, in order to create vaulted-roofed temporary architecture.

More images will be posted on: http://flatteringdomes.wordpress.com/



a b c

AutoCAD (a) and origami (b) models of a parabolic groin vault; the Icelandic church (c) that inspired our work, from http://goodchurchdesign.blogspot.it/2012/08/icelandic-stained-glass-by-gerur.html

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Experiments and Analysis of the Miura-ori Fold Core

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Abstract

Sandwich structures are often used in aircraft due to their high stiffness-to-weight ratio. Compared with the conventional cores, the origami inspired fold cores have a number of advantages such as the open ventilation channels, the continuous manufacture process, the superior energy absorption and impact properties, etc. In particular, the Miura-ori fold core has attracted researchers' interest in recent years [1, 2].

It is well known that the Miura-ori is a type of rigid origami, which can be considered as the mechanism if replacing the facets and crease lines with rigid panels and revolute hinges, respectively. Thus, the movement of the Miura-ori fold core can be described by the kinematics of corresponding mechanism [3].

In this study, copolymer Elvaloy is used to prepare the fold core through compression molding. The intrinsic mechanical properties of Elvaloy were obtained through tensile and 4-point-bending tests of standard samples. To eliminate the reinforced effect of the skins and bonding measure of the sandwich structures, the mechanical testing will be carried out on the fold core only which includes the newly introduced in-plane compressions in two principle directions (D1 and D2), the 3-point-bending compressions in three principle directions (D1, D2 and diagonal), as shown in Figure 1, and the flatwise compression. After each tests, the fold core specimen sprang instantly back with trivial changes of its original dimensions. Thus, the repeated usage of the fold core is possible.

To interpret the deformation and absorbed energy, an analytical model is proposed based on the kinematics of the Miura-ori and the mechanical properties of Elvaloy. With the analytical model, the geometries and materials of the fold core can be optimised to achieve the proper mechanical behaviour for certain applications, such as energy absorbing composites and protective gears.

Keywords: rigid origami, Miura-ori fold core, composite material

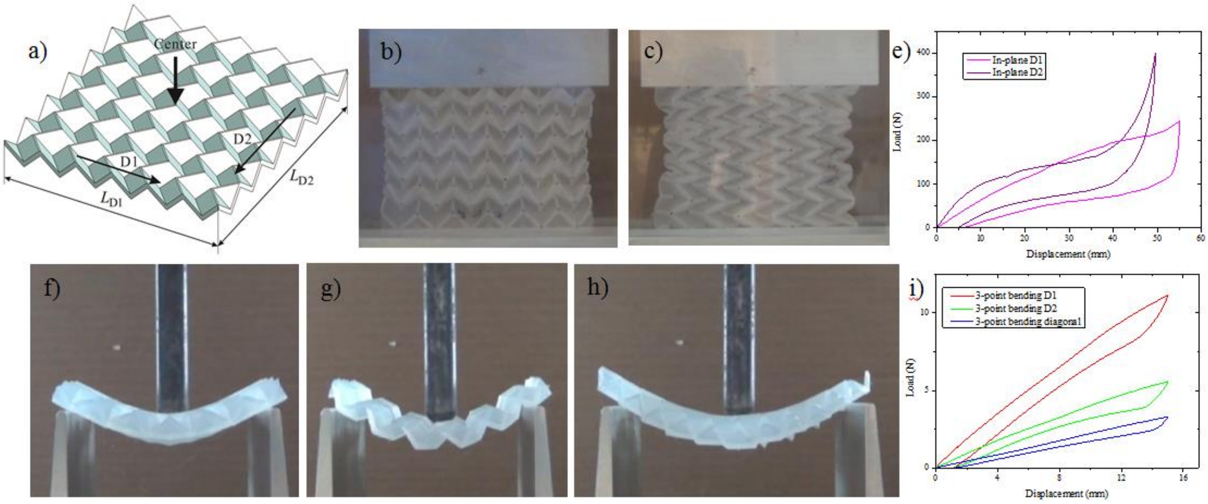

Figure 1. The set-up and results of the mechanical tests. a) Definition of direction D1 and D2. In-plane compression in b) D1, c) D2 c) displacement-load curve. 3-point-bending compression in f) D1, g) D2, h) diagonal and i) displacement-load curve.

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Area and Paper Optimization

Emma Frigerio (*) - Maria Luisa Spreafico (**)

At any school level, origami models are useful to introduce and explain mathematical concepts, especially geometric ones. Certainly, we could fold directly a geometric figure (such as a triangle or a rhombus) and study its properties; however, especially with younger students, often it is preferable to give a geometry lesson while folding a non-mathematical object (such as an animal or a plane). For instance, in her Origametria program Miri Golan uses this second approach.

In the same vein, during the past four years we have been teaching some lessons in Italian primary schools. To illustrate our work, we present a set of lessons, suitable for fifth-graders in the Italian school system, which takes children from an introduction of the concept of area and its measure to an optimization problem.

First, we use simple origami models to convince students that we need a unit in order to measure and compute areas. Then, while we teach them carefully chosen models, we invite students to measure areas of regions that naturally arise in the folding process. Since different students use different units, the subsequent discussion highlights important facts about measuring.

Finally, after teaching an easy folded envelope construction, we propose them an area optimization problem: “Find the most efficient paper size to produce an envelope of specified size”. In this last part, the main issue is not to discuss geometric aspects, which naturally come out of origami folds, but to investigate more analytic aspects. Students need to reopen the model, study its crease pattern, make a conjecture and verify it, both experimentally and analytically. This is a very good exercise, which allows students to work on a real-world math problem with a hands-on approach.

In each lesson, different skills are at stake, which keeps student motivation high; most importantly, each one can take advantage of the approach most suitable to her/his way of learning. This promotes a deeper understanding of the mathematical concepts involved and, we believe, a better long-term retention.

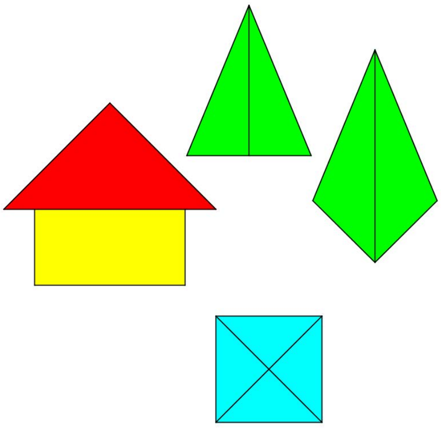

Figure 1
Comparing areas
with origami models

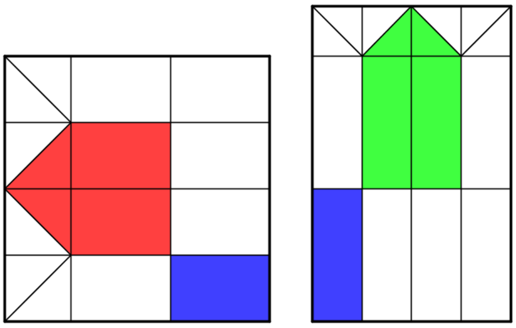

Figure 2
The optimization problem:
the rectangular sheet is more efficient

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Symmetric Colorings of Polypolyhedra

sarah-marie belcastro and Thomas Hull

Polypolyhedra (introduced in [1]) are compounds of edge-transitive 1-skeleta. There are 54 topologically different polypolyhedra, and each has icosidodecahedral, cuboctahedral, or tetrahedral symmetry; all are realizable as modular origami models with one module per skeleton edge. Consider a coloring in which each edge of a given component receives a different color, and where the coloring (up to global color permutation) is invariant under the polypolyhedron's rotational symmetry group. On the Five Intersecting Tetrahedra, the edges of each color form visual bands on the model (see Figure 1), and correspond to matchings on the dodecahedron graph. We count the number of such colorings and give three proofs. For each of the non-polygon-component polypolyhedra, there exist corresponding matching colorings, and we count the number of these matching colorings. For some of the non-polygon-component polypolyhedra, there is a corresponding visual-band coloring, and we count the number of these band colorings. Examples of each are given in Figure 2. We also investigate duality of colorings between polygon-component and non-polygon-component polypolyhedra.

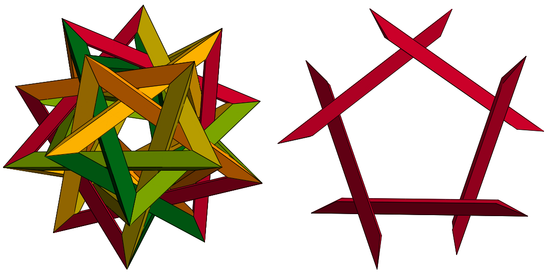

Figure 1: A symmetrically colored FIT (left) with visual bands (right).

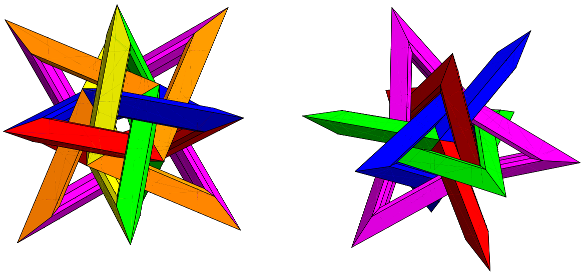

Figure 2: A 4-3-4 polypolyhedron with a matching coloring (left) and a visual-band coloring (right).

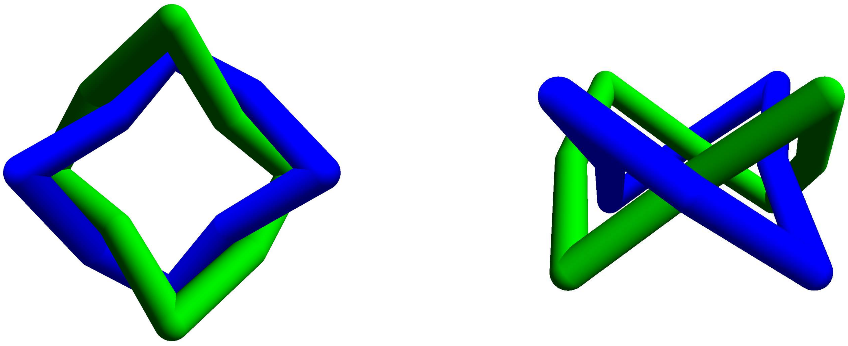

Figure 3: An edge-transitive (4,2) torus link seen from above (left) and from the side (right).

Finally, we show that there exist polypolyhedra with cyclic and with dihedral symmetry groups (see Figure 3 for one example); this disproves a claim of Lang in [1]. These lead to a set of torus link origami models.

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The Metamaterial Generated from Rigid-origami Pattern

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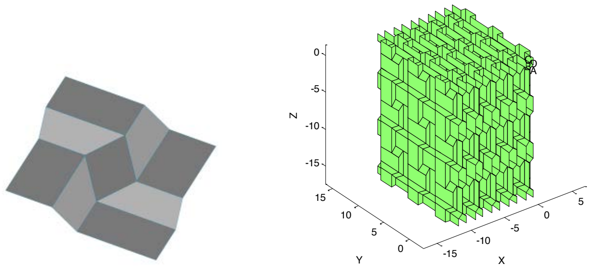
School of Mechanical Engineering, Tianjin University, Tianjin, China 300072

Abstract

Square-twist pattern has been found in arts, mathematics or engineering with different forms. Considering the distribution of hill and valley folds, there are four possible crease patterns. In order to study their rigidity, the mobile conditions and motion properties of the corresponding network of four spherical 4*R* linkages is analyzed. Although the corresponding network of four crease patterns are identical in the kinematic representation, but the motion of the four crease patterns in fact lies on different parts of kinematic path. Then two of them satisfy the kinematic compatibility condition of the mobile network of four spherical 4R linkages while the other two do not. So for the four possible crease patterns from square-twist pattern are rigid origami patterns while the other two are non-rigid ones.

Further study on extending the pattern unlimitedly following the tessellation method with these two rigid patterns shows that only one of them can form a tessellation of the identical pattern by interconnecting in both in-plane and out-of – plane manners. As a result, a type of 3D metamaterial is generated. When every single unit in the metamaterial folds and extends following the rigid motion, there will be a large deformation on the metamaterial. And due to the kinematic property of the single unit, the whole metamaterial exhibits negative Poisson's ratios in two directions. And the motion property of the pattern's folding dominates the metamaterial's structural mechanics. The unit with crease patterns and the corresponding metamaterial are shown in Figure 1.



(a) (b)

Figure 1 (a) Rigid origami unit; (b) the corresponding metamaterial.

Keywords: Rigid origami; Spherical 4*R* linkage; Assignment of hill-valley folds; Metamaterial; Negative Poisson's ratio.

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Crowdsourcing Origami Sculptures

Jeannine Mosely

In this paper, we present several projects where volunteers worked collaboratively to build very large origami sculptures by linking thousands of traditional business card cubes. These projects involved hundred of students and dozens of teachers/facilitators at dozens of schools over periods of months to years.

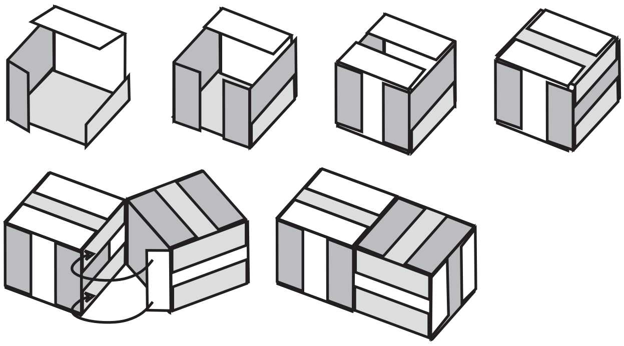

Fig.1 Building and linking business card cubes

We describe a variety of lessons in mathematics, engineering and aesthetics that can be taught through the medium of business card cubes and explain how to tailor the lessons to different grade levels. Mathematical concepts include simple arithmetic, exponentiation and logarithms, basic topology, Euler's formula, chirality and recursion. Engineering lessons include structural analysis, project planning and quality control. Aesthetic issues of color, scale and symmetry are considered.

In addition, we detail lessons that we learned about how to (and how not to) organize such large scale projects.

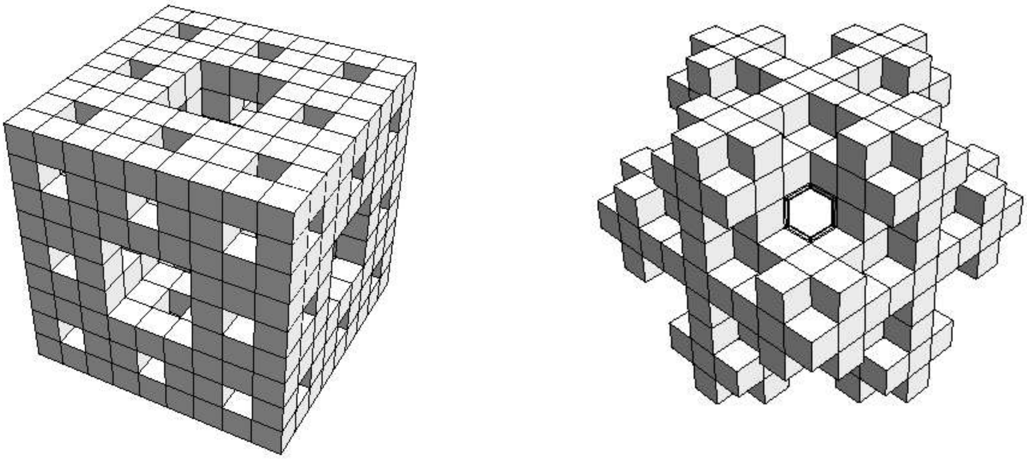

Fig.2 Two Fractal "Sponges" made from cubes

Origami for triangular spiral tilings

Takamichi Sushida (Meiji University)
Akio Hizume (Ryukoku University)
Yoshikazu Yamagishi (Ryukoku Uniersity)

Phyllotaxis is the arrangements of leaves and other organs of plants. In particular, it is well known that phyllotactic patterns of typical plants (e.g., sunflower) have structures with double spirals and they are intimately related to the golden section $\tau = \frac{1+\sqrt{5}}{2}$ and Fibonacci sequence.

As one of origami developments with a spiral folding structure, Tomoko Fuse devised the twisted multiple towers [1]. Also, as one of tilings, Akio Hizume devised triangular spiral tilings named Fibonacci Tornado, which have mathematical essences of phyllotaxis of seeds of sunflower [2]. As a mathematical feature of Fibonacci Tornado, they are tilings of the punctured plane $\mathbb{C}^* := \mathbb{C}\backslash\{0\}$ which admit a transitive action of a similarity transformation group generated by τ. Moreover, he made Fibonacci Tornado in an origami sheet, by applying a spiral folding structure of the Fuse's twisted multiple towers.

In this study, we consider a triangular spiral tiling of $\mathbb{C}^*$ with the vertex set $S = \{\zeta^j : j \in \mathbb{Z}\}$, which are determined by a suitable chosen triplet (ζ, m, n), where $\zeta = re^{\sqrt{-1}\theta} \in \mathbb{D}\backslash\mathbb{R}$ and $m, n > 0$ are relatively prime integers. When m, n are fixed, several generators ζ produce phyllotactic tilings related to the continued fractions of $\theta/2\pi$, or non-phyllotactic tilings. The set of generators ζ of phyllotactic tilings is the union of branches of real algebraic curves parameterized by θ. The set of generators ζ of non-phyllotactic tilings is the union of branches of real algebraic curves parameterized by r. In particular, it gives a dense subset of the parameter space $\mathbb{D}$. By considering shapes of triangles which admit spiral tilings, we obtain several examples of triangular spiral tilings and corresponding origami sheets (See Figure 1(a), (b)).

In this presentation, we show mathematical results for triangular spiral tilings and present origami sheets for triangular spiral tilings [3]. Moreover, by constructing a mathematical model of automatic expansion for rigid foldability, we show several examples of origami sheets with rigid foldability, for triangular spiral tilings (See Figure 1(c)).

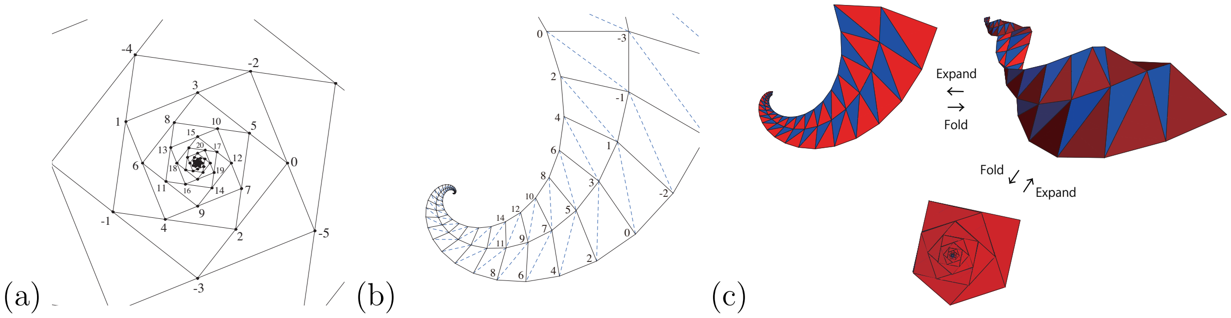

Figure 1: (a) A phyllotactic spiral tiling by right triangles with the corners $\pi/6$, $\pi/3$ and $\pi/2$. A generator is $\zeta = (0.92149\cdots)e^{2\pi\sqrt{-1}\cdot(-7/12)}$. $(m, n) = (5, 2)$. Each number $j \in \mathbb{Z}$ indicates a position of the complex coordinate ζ^j. (b) An Origami sheet of (a). (c) Automatic expansion simulation by a mathematical model of rigid foldability for (b).

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Mathematics and Art through the Origami of Cuboctahedron

Shi-Pui Kwan (spkwan@hotmail.com)

In 5OSME I had joined a workshop on Akari Origami. A Japanese lady introduced the construction a lovely model: a Checkered Lantern. Being a Mathematics and Physics teacher I recognized that it was a carefully selected LED lamp inserted into a folded model of a cuboctahedron.

This idea inspires and initiates my study in this solid. I have found various folding algorithms and have been amazed that it can even be constructed by business cards. Gradually I understand the reasons behind in selecting the particular folding method and specific type of paper for constructing the Checkered Lantern. And I start to ask myself: can it be done in other ways with materials easily accessible around us?

To be rigorous the Checkered Lantern is a cubohemioctahedron which is a faceted version of the cuboctahedron. These two geometric solids that are quite strange to primary and secondary students can be obtained from the dissection of a cube or an octahedron which are well known and familiar to them. Their volumes can be derived with various methods based on secondary mathematics. I realize that my studies can be integrated into the formal mathematics school curriculum.

Lanterns are popular amongst the Chinese. The Chinese Mid-Autumn Festival is also called the Lantern Festival. The cuboctahedron model can be developed into a safety lantern as a replacement of the traditional candle lantern. In this paper I would like to discuss some folding algorithms of the models, some elementary geometric analysis and some ideas of integrating this theme across mathematics, science and art in schools.

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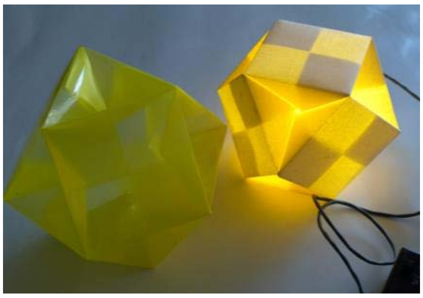

Checkered Lantern

cuboctahedron and cubic box

a folded model

Key words: cuboctahedron, mathematics, art

Polymer Sheets that Self-fold in Response to Light

Ying Liu, Russell Mailen, Yong Zhu, Alan Russell, Susan Brandeis, Michael D. Dickey, Jan Genzer

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We describe a simple approach to fold polymer sheets in a hands-free manner that uses localized light absorption on a pre-stressed polymer film to induce self-folding. Self-folding is a deterministic 'origami' process that causes a predefined 2D template to fold into a desired 3D structure with high fidelity. Self-folding takes advantage of the multitude of available 2D patterning techniques (e.g., lithography, inkjet printing, screen printing). Self-folding is attractive as a cost-effective 3D fabrication strategy for applications such as packaging, robotic actuators and sensors, biological devices, solar cells and reconfigurable devices. Most approaches to self-folding use hinges (i.e., regions of a substrate that fold) that have unique chemical composition from the bulk, which requires complicated photolithography or other multiple fabrication steps.

Our approach to self-folding relies on printing patterns of ink on a pre-strained polymer film (**Figure 1**). The substrates are commercially available pre-stressed polymer sheets. The hinges (black ink) are patterned using a desktop printer on either side of the optically transparent sheets. Hinges absorb selectively the light to heat the underlying polymer and causes the polymer sheets to fold into complex 3D structures (e.g., cubes, tetrahedrons) within seconds. The approach is appealing because it uses inexpensive materials and simple patterning techniques to form complex structures.

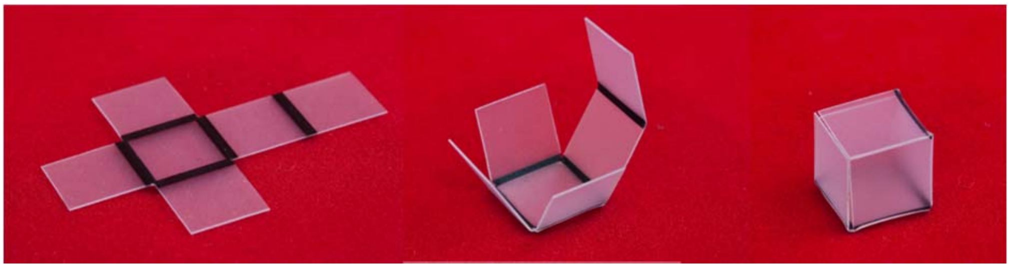

Figure 1. Demonstration of self-folding in response to light. The black 'hinges' are ink from an inkjet printer. When exposed to light, they get hot and cause the polymer sheet to fold due to strain relaxation of the polymer beneath the ink.

Our work is part of the Engineering Frontiers in Research Innovation (EFRI) on origami sponsored by the US National Science Foundation program. This special program has allowed us to build a team of engineers, artists, and mathematicians. We will describe our interdisciplinary work on self-folding origami. A video describing our work is on YouTube: "3D Object, Just Add Light".

An Assessment of Learning Geometry through Folding Paper by the Origametria Method, on the Visual Thinking of Grade Four Students

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John Oberman
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In our research, twenty Grade Four children from a school in the north of Israel learnt origami by the Origametria method once a week for 11 weeks. According to Van Hiele, there are five levels of geometric understanding, the first of which is Visual Thinking. Our research is focused on this level.

The lessons were observed, documented and photographed by student teachers and by teachers from the school. The answers given by the children during the lesson were noted and a chart made of their improvement. The conclusion of the research came from this knowledge.

The result of the research showed that the Grade Four children who studied geometry by making origami models by the Origametria method built for themselves an improved understanding of definitions and figures and had a reduced number of misconceptions.

In the light of our positive result, we propose two directives to support the teaching of geometry:

1. To strengthen a child's intuition and knowledge, and foster geometric thinking by folding paper.
2. That e-Learning support for teachers should include dynamic demonstrations (animations). In this way, teachers are released from the need to be specialists in origami, thus increasing the numbers of teachers who can participate.

Origami Teaching Re-imagined: The Kindergarten Origametria Programme

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The Kindergarten Origametria programme teaches the geometry curriculum of the Israeli Ministry of Education, to children aged 4-6 years. It was developed by Miri Golan, and is sponsored and monitored by the MoE.

The programme departs in several important respects from many conventions of mainstream origami teaching. Primarily, it does not teach named models, instead it permits each child to use his imagination when folding enabling him to vizualise and name models perceived in the geometry of the folds. In this way, and under the careful guidance of a trained kindergarten teacher, it uses the predisposition of a young child to learn primarily through imaginative play, rather than through instruction. Further, at each step of the folding, geometric investigation is undertaken by the children, in discussion with each other and with the kindergarten teacher. In this way, mathematic discourse develops and the use of mathematical concepts occurs.

The paper will outline the philosophy of the programme, describe the training given to kindergarten teachers, explain how the programme is taught in a kindergarten setting by kindergarten teachers and show what results are achieved.

The programme currently runs in approximately 100 state (public) kindergartens in Israel, primarily in Tel Aviv.

Paper Folding as a Topic of Design Education

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All designers fold. That is, all designers crease, pleat, bend, hem, gather, knot, hinge, corrugate, drape, twist, furl, crumple, collapse, wrinkle, facet, curve or wrap two-dimensional sheets of material, and by these processes of folding, create three-dimensional objects. These objects will perhaps not be origami-like in appearance, or the folding may only be a detail, but most will nevertheless have been folded wholly or in part, in some way. Since almost all objects are made from sheet materials, folding can be considered one of the most common of all design techniques.

Paul Jackson has taught 'Sheet to Form' workshops and projects since 1983, in more than 50 colleges of Higher Education across Europe, teaching students of Fashion, Textiles, Architecture, Ceramics, Product Design, Packaging Design, Fine Art, Jewellery and others. The paper will give an outline of his teaching methods, illustrated with examples of student work. It will also assess the value of folding as a creative practice in today's digital design environment and discuss why, in recent times, folding has become such an important vocabulary to professional designers.

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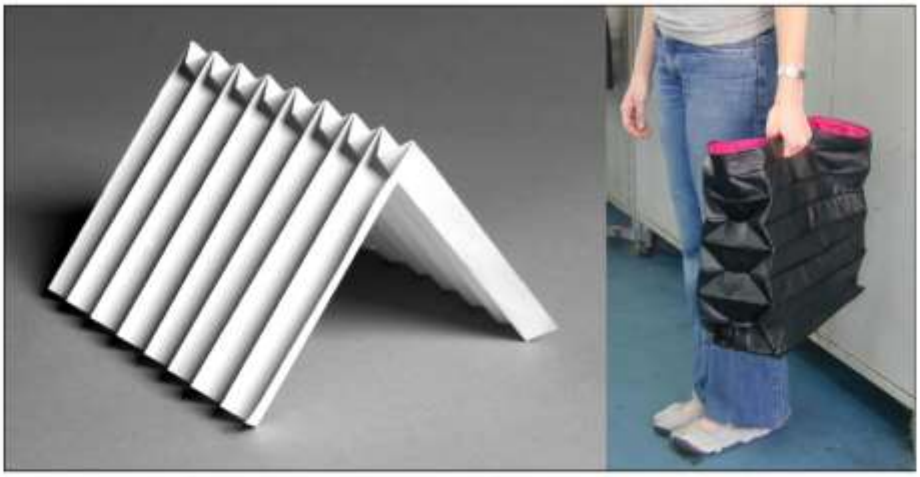

Bag
Leather, cotton

Deformed Polyhedral Skeletons

Shozo Ishihara

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Abstract

With a view to representing microscopic structures of crystals and molecules, polyhedral skeletons made of Origami had been assembled into three-dimensional structure models, named “Science Origami”. It had been demonstrated that Science Origami exhibits remarkable features not only as three-dimensional structure models of crystals and molecules in science but also as a teaching material in science classes in secondary education, as well as in higher one.

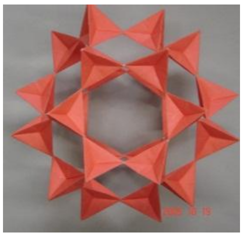
As most of polyhedra occurring in Nature are usually not regular but deformed ones, deformed polyhedral skeletons had been thought up by correlating shapes of triangular fragments constructing a polyhedral skeleton with sizes of sheet of paper. Recently, an Origami-model of Penrose Pattern made of only octahedral skeletons, which represent a quasi-periodic structure of the two-dimensional Penrose Tiling, has been invented, and it is suggested that the octahedral skeletons would correspond to golden octahedra appearing in the three-dimensional Penrose Tiling. It should be pointed out that the golden octahedron is actually deformed from regular one.

In the present meeting, some examples of deformed polyhedral skeletons made of Origami are exhibited, and the way how to make them is explained by focusing on the cases of golden tetrahedral and golden octahedral skeletons, which appear in the three-dimensional Penrose Tiling. Finally, it is concluded that Science Origami is applicable to any polyhedral skeleton.

Key words:

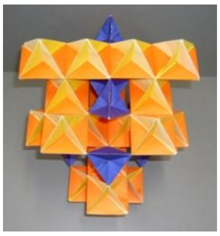
Deformed polyhedral skeleton, Golden tetrahedron, Golden octahedron

Science Origami:

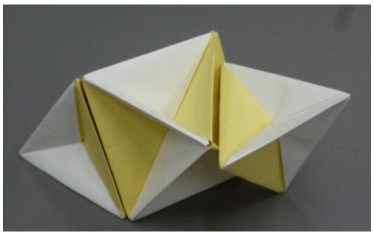


ゼオライト

ペンローズパターン



スピネル



黄金四面体と黄金八面体

General Folding Pattern Solution for Solids based on Polygonal Sections

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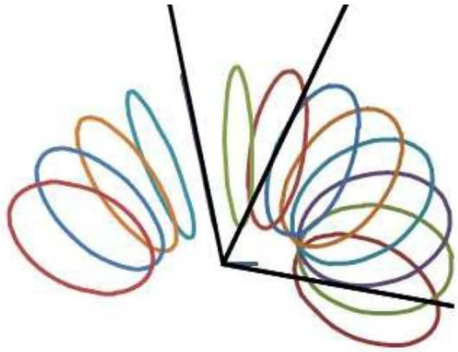
Keywords : Mathematics, Technology, Design

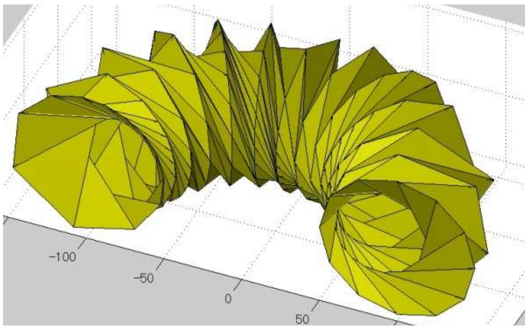
Abstract

To fold some three-dimensional shapes like a cylinder, cone, sphere, and torus, vertices and crease lines have to be calculated, satisfying two conditions, "Foldable condition at vertex (Kawasaki's theorem)" and "Flat-foldability". Conventional folding pattern solutions[1][2] use geometrical knowledge of the development of solid(ex. rectangles for cylinders, sectors for cones). The new method calculates a folding pattern based on a series of sectional polygons of a solid. To compare with conventional methods, this method enables to solve folding patterns for more kind of solid shapes.

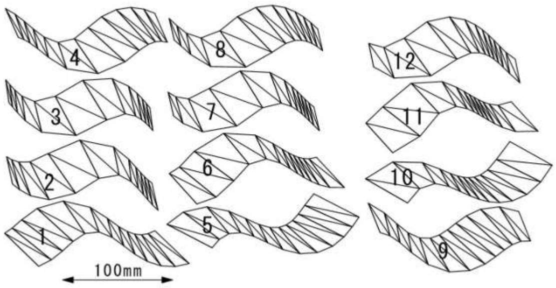
Examples of calculated and making tests.



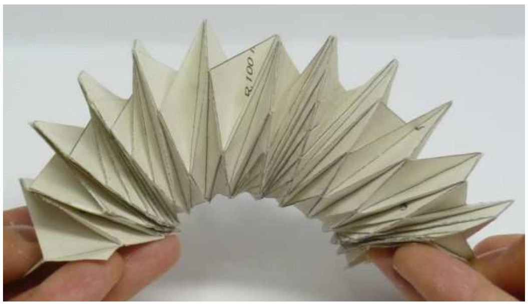
Modeling (Torus)



Calculate



Development



Making test

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[2] Development of Inflatable Parabolic Shell Using Origami Model. Takayuki Kusuda, Takatoshi Nojima, Atsushi Yuta. JPN (2002)

Flat-Foldable Axisymmetric Structures with Open Edges

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Keywords: Flat-fold, Axisymmetry, Kinematics

Some objects (such as cases or tents) have the important property of being flattened into a 2D configuration for storage purposes. These structures have a 3D shape that gives them some functionality, but can also be compacted when not used.

In this paper, we propose a method to design axisymmetric structures that can be folded flat with thickness. The flat configuration of the model is obtained by pushing the structure downwards. Our model is represented by a polyhedron with a vertical symmetry axis. The edges that are not coplanar to the axis are hinges that connect two faces allowing their dihedral angle to change. The other edges are open during the folding process, binding two faces only in the original 3D configuration.

The design interface consists in using mouse clicks to input an arbitrary outline formed by connected line segments. This outline is the desired axial cross section of the model. Usually, the input produces structures that are not flat-foldable and, therefore, we have to adjust the length and position of the line segments to enable flat-foldability (Fig. 1). This is an optimization process, changing the outline the minimum possible. In this process, we introduce evaluation values related with the folding process in order to make it easier by avoiding changes in the fold direction of hinges. Then, we add thickness to the optimized outline. Finally, we output the data of a flat-foldable model, the assignment of fold directions and the 3D animation of the folding process. (Fig. 2).

The results show that our design method is capable of generating flat-foldable structures. Figure 3 shows a real model constructed using the results shown in Figures 1 and 2.

Figure 1: Original outline input (black) and the modified flat foldable outline (red).

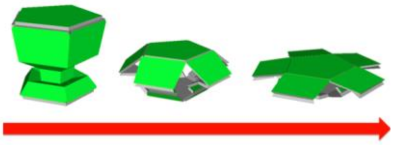

Figure 2: Animation of the folding process.

Figure 3: The real model built with 3D printer.

Simple Flat Origami Exploration System with Random Folds

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Category: Science
Keywords: Interactive system, Origami exploration, Random folds

Origami is a form of abstract art in which the artists label pieces of folded paper, relating the folded shape with another subject (animals and objects for example). Recent origami design techniques have made possible the design of realistic models. On the other hand, the action of labeling is still important for simple origami model, which is made with a small number of folds. In this paper, we propose an interactive system for exploring simple origami models by random generation of folded pieces. Our system generates origami pieces with *n* folds automatically and displays them, so that the user can focus on the labeling process. Every time the user requests another set of pieces, different shapes appear, because the generation process includes randomness. However, it is hard to generate a meaningful shape by completely random folding. For this reason, we use both folds with exact references and irregular folds (*Gurai-ori*) to generate new shapes. Origami folds can be parameterized using a set of reference points and/or lines. Although origami geometric constructions contain only combinations of regular folds, we can use partial references to limit irregular folds, but still leaving a degree of freedom (Figure 1). Our system uses both folds, and in the case of irregular folds, randomly determines the insufficient parameters. It also allows the exploration of origami shapes by similarity (Figure 2). If the user selects a set of folded pieces, our system generates another set, similar to the previous, by extracting the folds that are common to the items in the original set and composing them into new pieces.

The system is implemented as a web application. The users can register models they have discovered to a web database and share the diagrams that are automatically generated by the system.

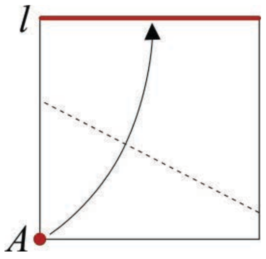


Fig.1 An irregular fold that places point *A* onto line *l*. There is one degree of freedom since *A* can lie in any point of the segment *l*.

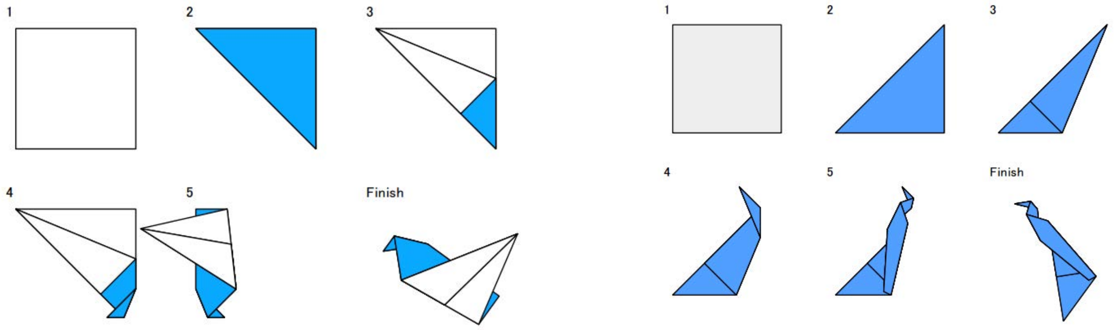


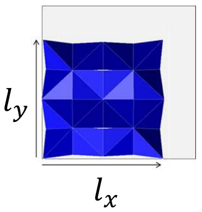
Fig.2 Two different bird diagrams with similar sequences. Both are generated by our system.

Physics-based Optimization of Origami Structures through FEM

Kazuko Fuchi*†‡, Philip R. Buskohl*†§, James J. Joo†, Gregory W. Reich† and Richard A. Vaia†

In recent years, the application of origami concepts to engineering design problems has become an increasingly active area of research. Fabrication simplification, efficient use of material, and reversible 2D to 3D morphing are a few of the motivations for this interest. Current approaches to integrate origami concepts are primarily geometric and focus directly on connecting target 3D shapes to a 2D pattern of folds. However, mechanical metrics, such as stiffness, deformation and energy, are also essential to explicitly include in the origami design process. To address this need, we propose the coupling of an optimization method with finite element (FE) mechanical analysis to design origami structures with desired performance, both geometric and mechanical.

$$\begin{array}{rl} \text{Find} & \{a_1, a_2, \cdots a_N\} \quad \text{that} \\ \text{Minimizes} & f = c_1 l_x + c_2 l_y \\ \text{Subject to} & g = \mathbf{u_0}^T K \mathbf{u_0} \leq E_{max} \\ & \mathbf{u_0} = \sum_i a_i \phi_i, \quad K\mathbf{u} = \mathbf{F} \end{array}$$



Compression is scaled up for demonstration

Figure 1: Optimization problem

We employ a modified frame element formulation that includes an explicit relationship between applied torque and fold angles as $\tau = \kappa(\theta_i - \theta_{if}) = \kappa\Delta\theta$. The stiffness associated with folding, frame elongation and flexure may be precisely scaled to reflect the design material's resistance to folding, in-plane stretch and facet bending. With appropriate scaling, we can identify and energetically rank the eigenmodes associated with folding versus those of stretch and facet deformation as in [1]; thereby isolating a set of folding motions to use as a perturbation from a flat configuration. Together with the mechanical and modal analyses, we can begin to investigate origami design goals beyond geometric shape matching. To demonstrate the utility of both the mechanical and modal analyses, we investigate the in-plane compression ratio of a modally perturbed 2D structure under a fixed in-plane load. Several objective functions are relevant for this design problem, such as minimal compressed area, maximal energy absorption, or target aspect ratio. We chose to minimize the projected y-length by selecting an optimal perturbation that consisted of a linear combination of eigenmodes specific to folding. A constraint on the mechanical energy of the initial perturbation is included to enforce efficient folding (see Fig. 1).

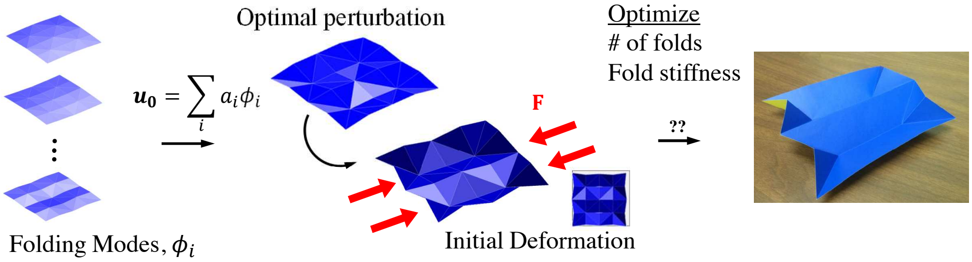


Figure 2: Optimal design

Figure 2 outlines the design process of finding the optimal perturbation for a symmetric structure loaded in-plane. In essence, the optimizer identified the folding directions for optimal compression along an axis orthogonal to the applied load. Further optimization on the number of fold lines or distribution of fold stiffness may reproduce a simplified version of this result (see paper model in Fig. 2). Future investigation will include the optimization of the initial fold pattern to broaden the design space of viable origami structures and eliminate unnecessary foldlines.

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Mechanics of Miura-ori Lattice Defects

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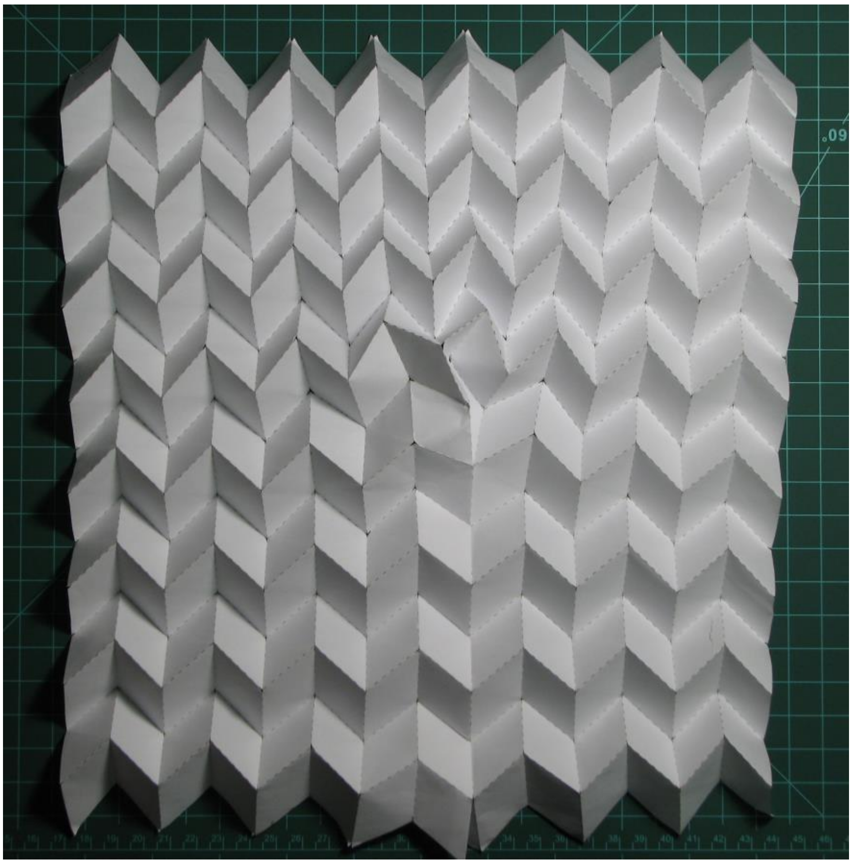
The mechanical properties of origami-inspired materials show remarkable potential for responsive, tunable next-generation materials. For example, the Miura-ori fold is predicted to have negative Poisson ratio and anisotropic compressive properties. Using a custom mechanical testing device and 3D laser profilometry, we investigate the moduli and the role of curvature in setting these material properties. Because defects are known to dramatically alter the bulk properties in other periodic materials, we introduce defects into the folding pattern to investigate their effects on the macroscopic mechanical properties. Interestingly, we find that a single defect increases the overall material stiffness, but the introduction of a second defect in the opposite direction can cancel out the first, tending to restore the original material properties.



Mechanics of Snap-Through Transitions in Twisted Origami

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Chris D. Santangelo,[3] Ryan C. Hayward,[2] Itai Cohen[1]

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[2]Department of Polymer Science and Engineering, University of Massachusetts, Amherst, MA 01003, USA
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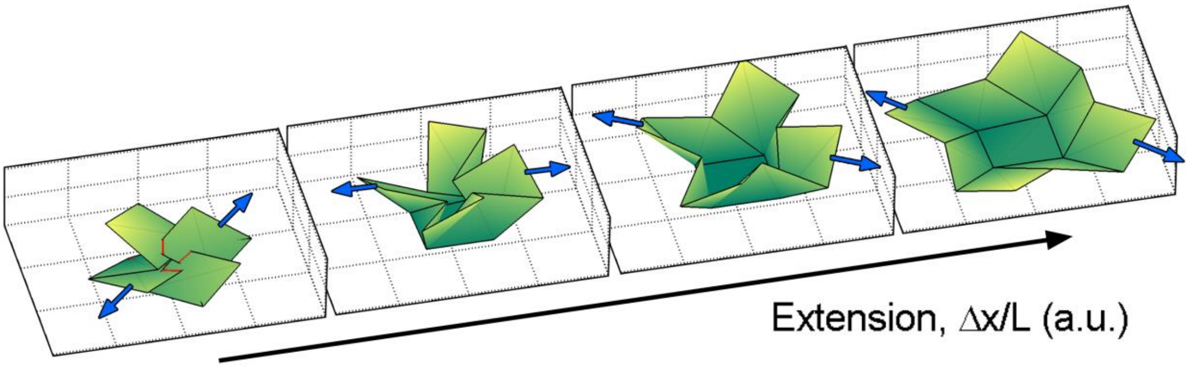
In folding origami patterns, one frequently encounters situations where the next step requires the model to pass through an extremely contorted configuration that places an unusually high stress on the paper. If the material does not tear, the final state generally relaxes these internal stresses, while the “snapping-through” transition locks the folding configuration in place. Here, we experimentally investigate this transition between configurations in tessellated folding patterns using a custom built mechanical testing device. By varying the folding angles, we are able to move from a pattern that exhibits snap-through transitions to a pattern that does not. Borrowing from the language of thermodynamics, we interpret these results in terms of phase transitions and the topology of allowed configurations in phase space.



Origami and Teaching Language and Composition

By Lainey McQuain and Dr. Alan Russell

Origami is used as a teaching tool in STEM classrooms; and most applications of it may be found in math classrooms. When using origami as a pedagogical tool, it would seem unfair to limit its use to only a few core subjects. It is important to consider how origami is beneficial in other areas. Origami can be used in the English classroom, specifically to teach writing (language and composition). Paper-folding provides a creative outlet and storytelling module within the English classroom, but there is limited work done that considers origami as an academic tool for teaching English. Lillian Yee Ho conducted research that combined origami with teaching English to English language learners (247). This is a point of origin for how origami was first used as a tool for writing support in the English classroom.

Research around using origami to teach language and composition is nearly non-existent, but there is a plethora of research revolving around the idea of visuals in writing. Diana George suggests, "…A common theme about visual communication that might be summed up in this way: Literacy means more than words, and visual literacy means more than play" (16). George's argument can be directly tied back to Yee Ho's work with adult English language learners and origami. Yee Ho is one of few researchers who have applied origami to language and composition studies, but her work has found that using origami as a visual can promote cooperative learning as students try to verbally explain how to fold a model or even practice their written instructions. Yee Ho's most successful practice was the use of a fortune-teller model to help English language learners grasp English sentence construction (249). There are also generated activities specific to the use of origami and origami visuals in the writing classroom without the implications of ESL learners (ex: storytelling/narrative writing with origami models) (252). Writing is a communication tool made up of literacy in words and visuals. The two cooperate with each other, and students are more successful culturally and artistically when the two are combined. Using origami in conjunction with teaching writing/language and composition seems to be a logical extension of the research already conducted to support the use of visuals in the writing classroom.

In addition to Yee Ho's work and the support for communication using visuals and writing, Common Core State Standards emphasizes that students need to initiate and participate in a range of collaborative discussions. As visuals and writing are combined, students are engaged and begin collaborative discussions. Thus, this research focuses on CCSS.ELA-Literacy.SL.9-10, which specifies that students will initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. This standard may also be applied to the secondary education grades 11-12. This standard was kept in place as the rationale for introducing origami as a way for students to improve their language and composition skills.

For this research, qualitative methods were used. Participant observation, interviews with participants, collection of participant writing, and various sources of non-written data from the participants were used to determine research results. Interview questions were open-ended and were aimed at allowing the participants to discuss their experiences without limitation of direction.

Results were collected from two groups of students; these students were a mixture of beginner to moderate paper folders. In order to determine the success of origami on language and composition, two classes of *undergraduate* students were asked to adhere to two Common Core State Standards: CCSS.ELA-Literacy.SL.9-10.4 present information, findings, and supporting evidence clearly, concisely, and logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task; and CCSS.ELA-Literacy.W.6.2d use precise language and domain-specific vocabulary to inform about or explain the topic. To summarize, students were given diagram-only instructions of several beginner-level origami models and asked to write English-only instructions for the diagrams. Students were to be specific and concise and were given a list of origami terminology to use. Ultimately, their instructions had to be 10 steps or less.

Our research has found that when applied to these 3 specific Common Core State Standards, origami has the potential to introduce students of various grade levels to technical writing. By breaking out of storytelling, Lillian Yee Ho discovered that origami could benefit the teaching of English sentence construction. Yee Ho found that origami had multiple uses in the English classroom, and our research has found that challenging students to combine origami visuals and writing has several outcomes. Students are able to practice writing that considers a specific audience in order to address the purpose of providing that audience with appropriate instructions; and students are able to recognize that unclear and inappropriate written instructions cause communication problems between visuals and writing.

When origami is applied to the area of technical writing, it allows for students to recognize that well-written instructions can either make or break communication. Origami in education is not just limited to STEM classrooms; it may be successful in other subjects. Its use of diagrams and written instructions allows teachers of language and composition to now consider alternative methods of teaching technical writing to students who are constantly surrounded by a culture made up of the mass use of visuals combined with writing/rhetoric.

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ORI* On the Aesthetics and Language of Folding and Technology: Scale, Dimensionality, and Material

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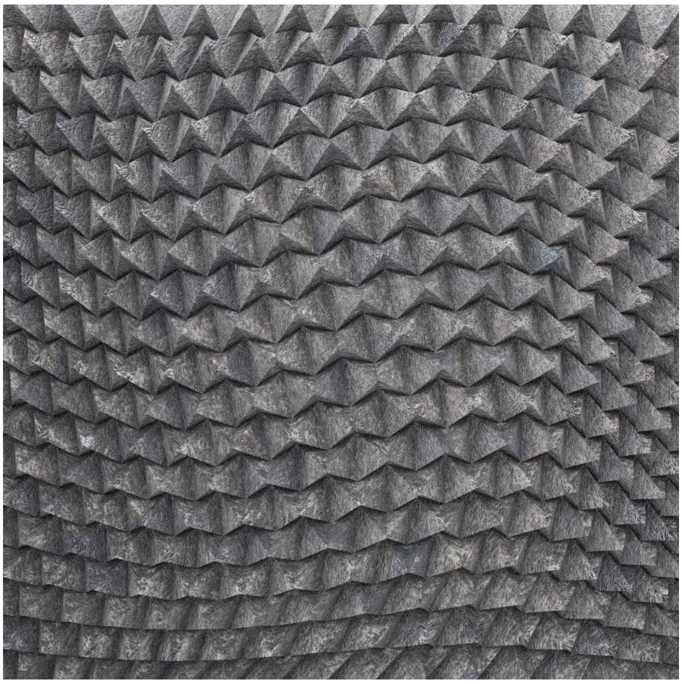
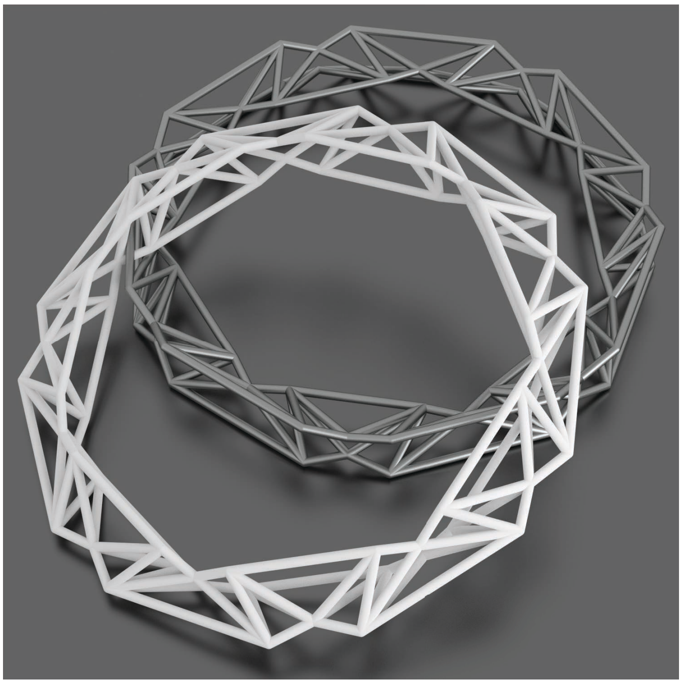
The model for the sciences of matter is the 'origami' as the Japanese philosopher might say, or the art of folding paper. – Gilles Deleuze, The Fold, Leibniz and the Baroque.[1]

A deceptively simple question such as "What is a fold?" produces simple and complex answers. Origami provides us a with a simple answer that we can demonstrate; make a crease in a sheet of paper and there it is, done. This answer, however beautiful, points to, but does not address the infinite possibilities of folding that origami artists, scientists, mathematicians and educators know and relish, nor does it even begin to explain folds in DNA (Dioxyribonucleaic Acid) chains[2], pleats embedded in high fashion[3], creases in the facades of new architectural icons[4], or theories for the shape of dark matter[5]. ORI* (meaning: fold anything) is the umbrella title of new research by artist Matthew Gardiner, that examines origami, and more specifically folding (ORI/ORU, 折り/折る), through a philosophical lens. A preliminary theory is that folding is a language of structure, a means of programming nature and materials, allowing the design of functional and aesthetic forms in manmade and natural systems. This paper will present a facet of this research: as an investigation into scale, dimensonality, and material within the following scale domains: the nanoscale, origami scale, interior scale, exterior scale and space scale. The paper will present actual and conceptual experiments that combine the artistic logic of symbols, scientific methodology, and fabrication technologies.



Feltfold: Rhino/Grasshopper generated panellisation of a waterbomb base across a surface of lofted curves.



Folding laws: 3D printed jewelery, a conceptual study of nanoscale chains using panellisation over a toroid.

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Spiderwebs, Tilings, and Flagstone Tessellations

Abstract

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Origami tessellations are an ever-expanding field of origami that produces beautiful artwork and has practical applications in fields of architecture, structural forms, and deployable structures. In previous work with Alex Bateman, we showed that a necessary and sufficient condition for the shrink-rotate algorithm to create a simple flat twist origami tessellation was the existence of a non-crossing reciprocal diagram for the tiling (a concept introduced by James Clerk Maxwell). The reciprocal diagram, if it exists, is non-crossing if and only if the tiling is a spiderweb, i.e., a graph that can be tensioned in such a way that all edges are under tension, rather than compression or slack.

In this work, I consider the desirable class of origami tessellations known as flagstone tessellations, in which the tessellation is fully flat, every tile is represented by a polygon surrounded by mountain folds and all such polygons meet edge-to-edge. The tessellation construction unit is more complex than the shrink-rotate algorithm for a simple flat twist, but there is still a simple geometric construction for all creases from the original tiling and the reciprocal diagram. I show that the spiderweb property is necessary to achieve a non-crossing flat-foldable crease pattern, but it not sufficient, and I provide sufficient conditions for existence of the desired pattern. Examples of flagstone tessellations and connections to related algorithms, e.g., Tachi's *Origamizer* and twisted flagstone tessellations, will be discussed.

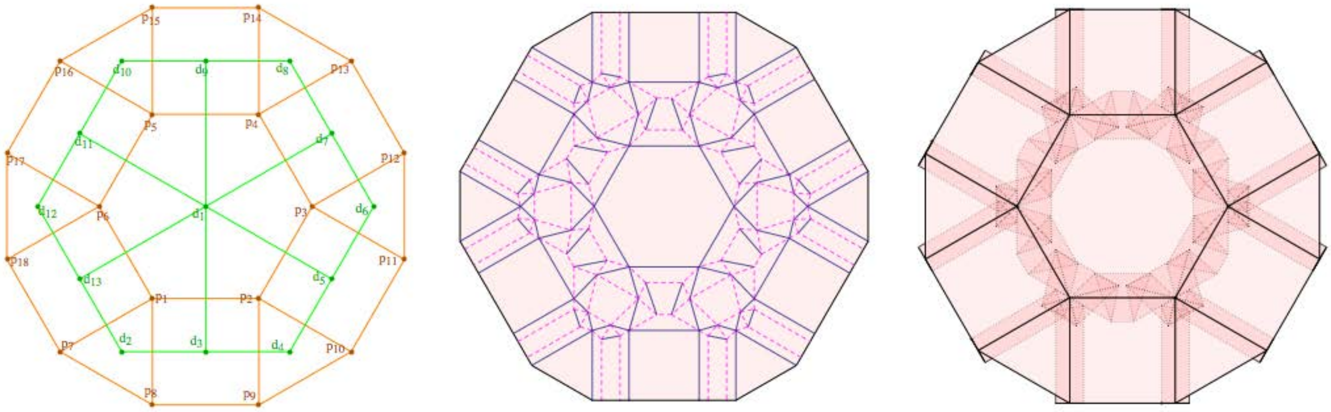

Figure 1. Left: the tiling (orange) and its reciprocal diagram (green). Middle: the computed crease pattern. Right: the folded form (translucent view, showing hidden layers).

Master Peace: An Evolution of Monumental Origami

Abstract

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Origami is traditionally thought of as being made of folded paper, as ephemeral and fragile, as existing only at the handheld scale. However, the art form has expanded to encompass other materials, other sizes, and has incoporated themes and concepts that arise from a wide range of influences. In this paper, we will present some of our collaborative large-scale/monumental origami sculptures and their evolution from the traditional paper-based art to monumental works cast and fabricated in bronze and stainless steel at considerable size and scale.

We use three distinct techniques for rendering origami in metal. First, 1-1: direct casting from paper and wax originals. This method precisely captures the surface texture of paper down to the watermarks and is usually used on indoor pieces. Second, monumental: We laser scan the paper/wax original, digitally enlarge, and mill with resculpting in foam/clay. This method captures the essential shape of the piece for large outdoor works. Third, heroic: We deconstruct the exterior shape and enlarge the work via Risa, Tekla software and fabricate individual facets from plate metal. This method replicates the essential shapes of the original for viewing from a distance while surviving sustained wind loads, earthquakes, etc.

We will present several case studies: first, the development of a 24-foot origami *Pegasus*, whose design evolution begins in 1960s-style Bird-Base-based forms but evolved through several different origami bases and then ultimately, was deconstructed into a series of stainless-steel fabricated shapes that will be reconnected on its final pedestal. Second, we present *Master Peace*, an *hommage* to the story of Sadako and the 1000 cranes, a stainless steel cast *tsuru* sculpture in which the cranes exist in both reflection and physicality. Along the way, we will present several other examples of origami translated into bronze and steel, at scales ranging from desktop to heroic.

Figure 1. The genesis: *Dancing Pony*, by Kevin Box (with T. J. Fu).

Figure 2. Evolution and hybridization in paper.

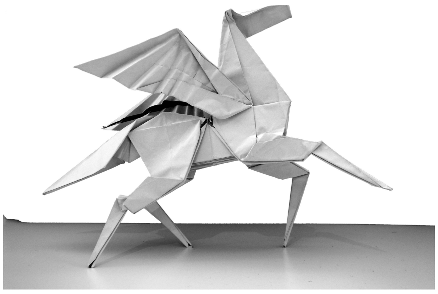

Figure 3. The origami *Pegasus*.

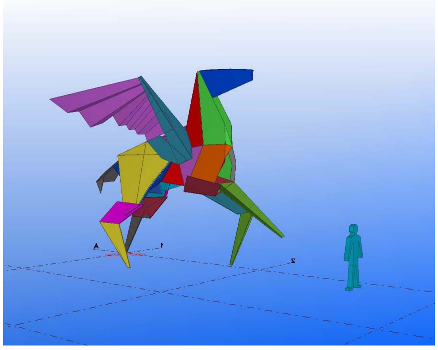

Figure 4. Heroic scale design.

Figure 4. Intermediate-scale model of *Master Peace*.

Figure 6: The cranes of *Master Peace*.

Origami Graph Paper for Polygon Packing and One-Straight-Cut

Abstract

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Early algorithms for origami design included the circle-packing techniques developed by Lang and Meguro in the 1990s [2, 4]. Other folding problems such as the one-straight-cut problem were solved by Demaine et al [1]. A connection between these problems arose with the development of polygon-packing [3] as a refinement of circle-packing; both algorithms utilized the straight skeleton as an element of the construction. They also shared a deficiency: the possibility of regions of space in which parallel creases lie arbitrarily close together.

Lang showed that for polygon packing, dense creases could be avoided by confining the edges of packing polygons to a periodic grid of squares or equilateral triangles. Such a grid works equally well to tame the one-straight-cut problem by confining cut lines to the same grid. This left open questions: are there other periodic grids that have the same desirable property of guaranteeing finiteness of the constructed symmetry pattern? Are any such grids, if existent, practical and useful?

In this paper, we answer both questions in the affirmative. We analyze the problem of finding multiply periodic grids of lines that have the property that every possible polygon-packed origami design has a finite isometrically flat-foldable crease pattern. We show that an infinite family of such grids exist and provide an enumeration method. Each grid can be characterized by a figure of merit that corresponds to the minimum crease spacing in a design. Ranked by this FOM, box-pleating is most efficient; hex-pleating comes next; but there is a new grid whose FOM is comparable to the former two that, to our knowledge, has not been described before and is of practical utility. We present these grids and will comment on related questions of interest, such as the feasibility of a grid that supports 22.5°-symmetric designs.

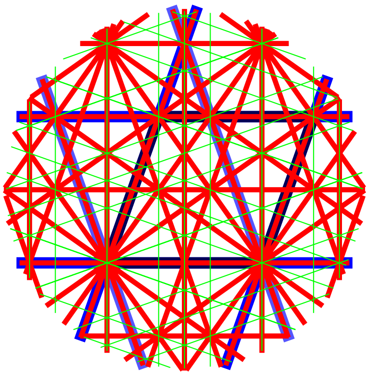

Figure 1. A patch of the new grid. Hinge lines are dark blue; ridge lines are red; axial lines are green.

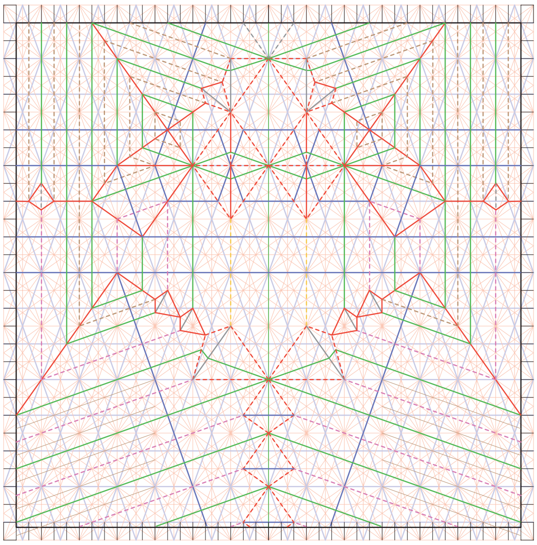

Figure 2. Crease pattern constructed on the new grid. Vertical and horizontal tic mark spacing is in the ratio $1 : \sqrt{2}$.

Figure 3. The origami figure designed on the new grid.

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Pentasia: an Aperiodic Origami Surface

Abstract

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Modular origami is commonly used to construct origami polyhedra. While there exists a vast literature of modular origami forms, the great majority of these are based on a relatively small number of basic forms: typically the Platonic or Archimedian solids, with variations in surface decoration and/or in the structural relationship of the origami unit.

In a recent work [1], we suggested that aperiodic patterns, such as the well-known Penrose tilings, showed great promise as a subject for modular origami development. In particular, we introduced the aperiodic 2-D surface invented by John H. Conway that he titled "Pentasia" and we showed how it could be realized as a modular origami structure.

The same features that make aperiodic surfaces interesting for modular origami also make them suitable for single-sheet folding. In this work, we present the Pentasia surface and explain its relationship to the Penrose tiling, giving inflation/deflation rules for its generation to arbitrary size. We present a modular unit that can realize the form. We then turn to the problem of rendering such surfaces using single-sheet origami, and show how the 3D surface can be decomposed into a crease pattern tiling whose unit cells are the dual graph of the underlying Penrose tiling. Using this technique, a single-sheet crease pattern can be realized to fold an arbitrarily large section of the Pentasian surface. The decomposition technique has connections to other computational origami algorithms ranging from tucking molecules of *Origamizer* to the universal molecules of tree theory .

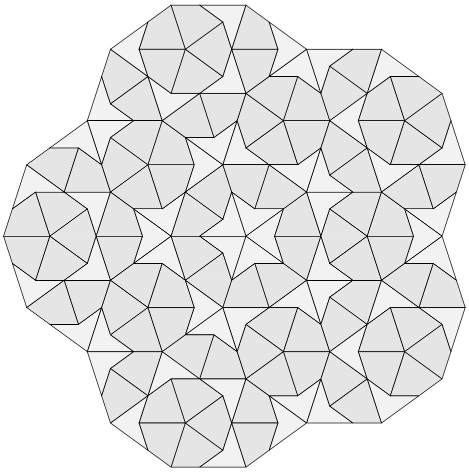

Figure 1. A kite-dart Penrose tiling.

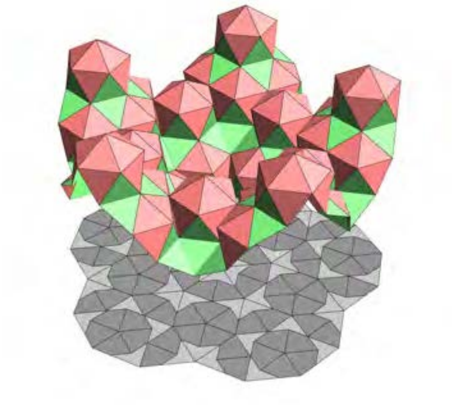

Figure 2. The Pentasia surface corresponding to the tiling of Figure 1.

Figure 3. A portion of the Pentasia surface rendered in modular origami.

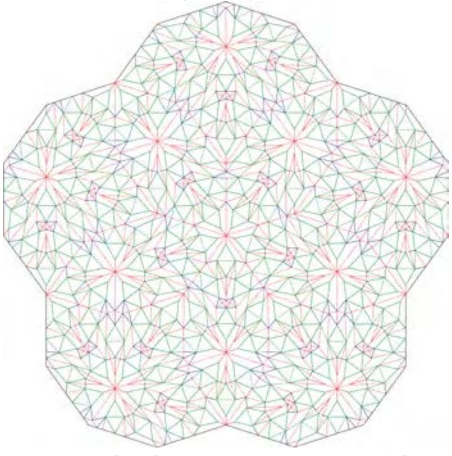

Figure 4. A single-sheet crease pattern of the surface shown in Figure 2.

References

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A Graphical Technique for Designing Rigidly Foldable Origami Mechanisms

Abstract

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We present a graphical technique for designing and analyzing rigidly foldable origami patterns composed of degree-4 vertices. The technique relies on a mathematical property of flat-foldable degree-4 vertices: in all states between unfolded and fully folded, all four dihedral half-angle tangents have a common coefficient of proportionality that depends solely upon the sector angles of the vertex and not upon the degree of openness of the vertex. For a given mountain/valley-assigned crease pattern, we construct a weighted digraph on the creases wherein the weights capture this constant of proportionality and the edge directions capture the relative inequality of dihedral angle magnitudes. The resulting digraph therefore describes relations between their respective fold angles that apply in every partially folded state that can be used to determine rigid-foldability. We call this weighted digraph the *multiplier graph* for the crease pattern.

From the multiplier graph of a given crease pattern, questions of rigid-foldability are easily addressed by considering simple structures in the graph. For example, the presence of a cycle establishes lack of rigid foldability. The presence of multiple graph dipaths between two creases establishes conditions on sector angles that permit solutions for rigidly foldable patterns in special cases. Using this technique, we have identified several new rigidly foldable patterns that have not been previously described and have established the infeasibility of other classes of patterns that are locally rigidly foldable. In particular, for simple flat twist folds, we show that for arbitrary polygonal twists:

(1) Every cyclic simple flat twist is non-rigidly foldable;
(2) Every alternating simple flat twist is non-rigidly foldable.

We further show true rigidly foldable simple flat twists for other twist crease assignments. Such patterns may be used as building blocks for deployable structures and other multistate mechanisms.

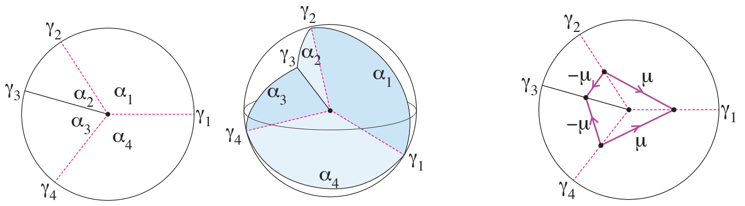

Figure 1. Left: schematic of a general flat-foldable degree-4 vertex, showing notation for sector angles $\{\alpha_i\}$ and dihedral angles $\{\gamma_i\}$ in the unfolded state. Middle: same for the partially-folded state. Right: The multiplier graph on the vertex. The edge weight is the ratio of the dihedral half-angle tangents (see next). The edge arrow always points from angles with larger magnitude to those with smaller magnitude, a relationship that holds for any non-flat configuration.

The sector angles and dihedral angles are related by the following equations at all positions between flat and fully folded. We define μ to be the scalar *multiplier* for the vertex.

$$\frac{\tan\left(\frac{1}{2}\gamma_1\right)}{\tan\left(\frac{1}{2}\gamma_3\right)}=-1,\ \frac{\tan\left(\frac{1}{2}\gamma_2\right)}{\tan\left(\frac{1}{2}\gamma_4\right)}=1,\ \ \mu\equiv\frac{\tan\left(\frac{1}{2}\gamma_2\right)}{\tan\left(\frac{1}{2}\gamma_1\right)}=\frac{\tan\left(\frac{1}{2}\gamma_4\right)}{\tan\left(\frac{1}{2}\gamma_1\right)}=\frac{\sin\left(\frac{1}{2}(\alpha_2+\alpha_1)\right)}{\sin\left(\frac{1}{2}(\alpha_2-\alpha_1)\right)}.$$

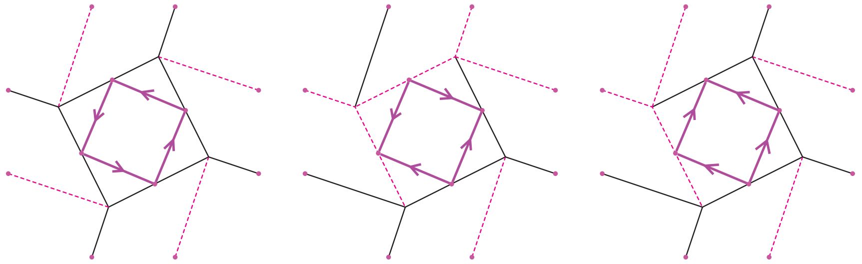

Figure 2. Multiplier graph on (left) cyclic (M^4) twist, (middle) M^2V^2 twist (seen in Barretto tessellation and generalized Miura-ori), (right) M^3V twist. The cyclic twist is not rigidly foldable because its multiplier graph contains a cycle. Both of the others are rigidly foldable and therefore may be used as building blocks in rigidly foldable origami mechanisms.

Tessellatica: A *Mathematica* System for Origami Analysis

Abstract

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Over its 25-year history, Stephen Wolfram's *Mathematica* software has found application in a wide range of fields from pure mathematics and physics to engineering, economics, and the arts. It combines symbolic and numerical analysis and supports a wide variety of programming styles, including procedural, functional, and object-oriented programming and contains strong support for output and 3D visualization. These properties make it particularly desirable for exploratory work in a mathematical field such as origami: one can carry out algebraic derivations and analysis, carry out numerical computations (up to and beyond constrained optimizations), and create high-quality 2D and 3D renderings of the structures under consideration.

It is common to develop a Mathematica package containing function definitions and subroutines to support a particular field; one of the better-known of these is *Combinatorica*, a package that supports modern graph theory (now distributed with *Mathematica*). In this work, I present a package for origami, called *Tessellatica*, which, as its name suggests, is particularly suitable for origami tessellations, but that in fact has applicability to a wide range of origami structures and problems. I will present the structure and underlying model of *Tessellatica 2.0*. Among the objects *Tessellatica* constructs, manipulates, and visualizes are: single 2D and 3D vertices, arbitrary crease patterns, computation of their folded form in 2D (with hidden line removal), computation of folded form in 3D, and routines for solving for crease pattern/folded form pairs for many types of tessellation, rotationally symmetric 3D forms (pots), and more. The nature of such *Mathematica* packages is they are open-source and readily extensible; I anticipate a public release of *Tessellatica 2.0* by the 6OSME conference.

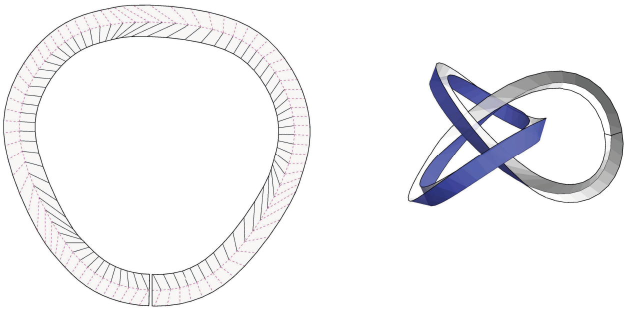

Figure 1. Computed crease pattern that folds into a trefoil knot.

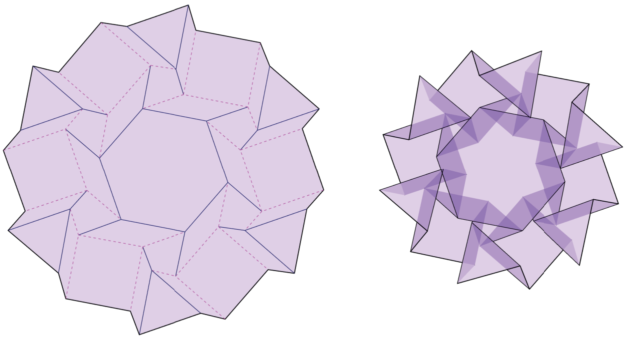

Figure 2. Crease pattern and computed folded form of a 3.4.3.6 tessellation with hidden-line removal.

Figure 2. Two different states of folding of a Huffman grid (quadrilateral grid of identical degree-4 vertices).

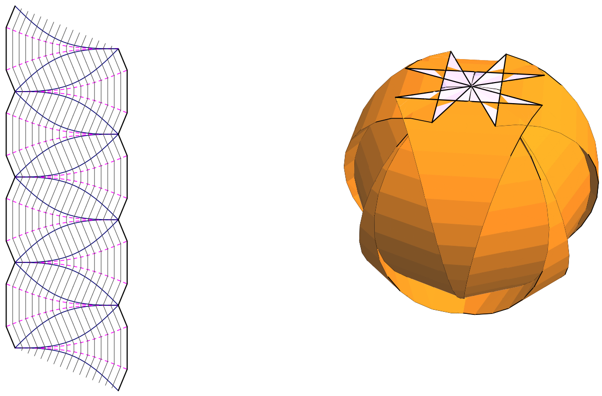

Figure 4. Computed crease pattern and folded form of a rotationally symmetric alternating-flange orb (from a concept by Jeannine Mosely).

Locked Rigid Origami with Multiple Degrees of Freedom

Zachary Abel Thomas Hull Tomohiro Tachi

In many practical applications of origami using hard or thick material such as metal and cardboard sheets, origami is well-modeled as rigid origami, i.e., a rigid panels and hinges model, which does not allow for material bending and traveling creases. The continuous foldability of rigid origami from a flat unfolded state to the folded state is essential for enabling the folding-based manufacturing process to make various 3D forms and tessellations.

It is known that a lot of origami models are not rigid-foldable from the unfolded state, because there exists a “locked” state which cannot be rigidly unfolded. We tend to explain such a lock as being attributed to the collision of facets or the lack of degrees of freedom. In fact, many locked rigid origami can be continuously unfolded in kinematic simulations if the polygonal mesh is triangulated to have multiple degrees of freedom and we ignore collisions so that the facets can penetrate each other. This observation leads to a hypothesis that any triangulated mesh origami can be continuously folded from an unfolded state. However, in this paper we show that a triangulated crease pattern is not enough to make it rigid-foldable by constructing a locked rigid origami from a triangulated crease pattern. The proposed rigid origami is a developable triangular mesh with 6 boundary edges (thus with 3 degrees of freedom) and yet has a folded state which cannot be continuously unfolded to a flat sheet of paper even if faces are allowed to penetrate each other (Figure 1).

We investigate the full three-dimensional configuration space of the proposed model, and prove that the configuration space is comprised of two disconnected domains. We further parameterize the pattern by a sector angle θ and explore its family to find out the critical patterns at which the configuration space changes its topology.

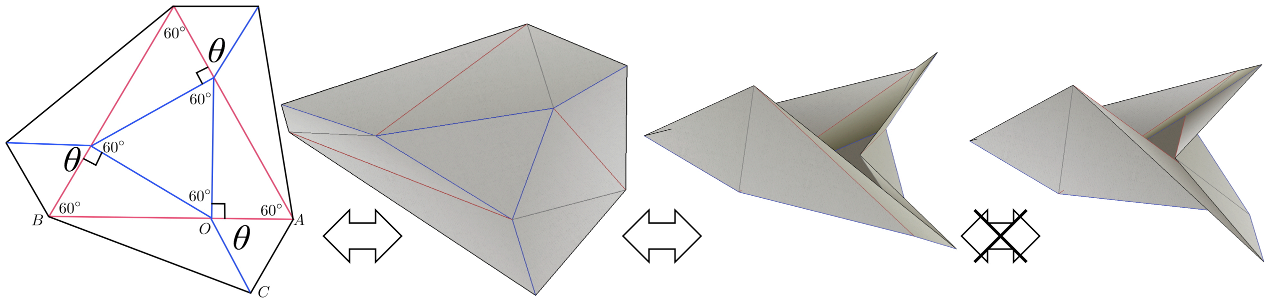

Figure 1: The crease pattern (left), continuous folding (middle two), and locked folded states (right) of the proposed pattern. Red and blue lines indicate mountain and valley creases respectively. θ is the parameter that changes the connectivity of the configuration space; in this figure θ is set to 62°

Cell Origami : Producing 3D tissue using origami folding technique

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I will demonstrate that self-folding of three-dimensional (3D) cell-laden microstructures by using cell traction forces and origami folding technique on a massive scale (Fig. 1a). The benefits of culturing cells on 3D microstructures have been widely acknowledged. One of the essential requirements for a method of fabricating 3D microstructures with cells is that they can be produced simply on a massive scale. Previously, I have demonstrated that cell traction force can be used to drive origami-like self-folding of flat cell culture plates into various 3D microstructures. However, the microstructure could only be produced one by one. In this paper, I have achieved control of the folding angles to produce 3D cell microstructures with precise shapes on a massive scale. Cells are selectively cultured and stretched onto flat microfabricated plates with a flexible joint that is located between the plates. The microplates spontaneously lift and fold up into 3D microstructures due to the cell traction forces. The folding angle is determined by the dimensions of the flexible joint. I successfully constructed diverse 3D cell-laden microstructures on a massive scale (Fig. 1b). Using the self-folded cell-laden microstructures, I also assembled various 3D cell structures manually, and furthermore by cell own migration: I found that the cells sprouted through the self-folded microstructures, and the structures migrated and rotated. Consequently, the microstructures self-assembled into various shapes. This building approach promises be a useful tool for various 3D tissue culture and engineering applications.

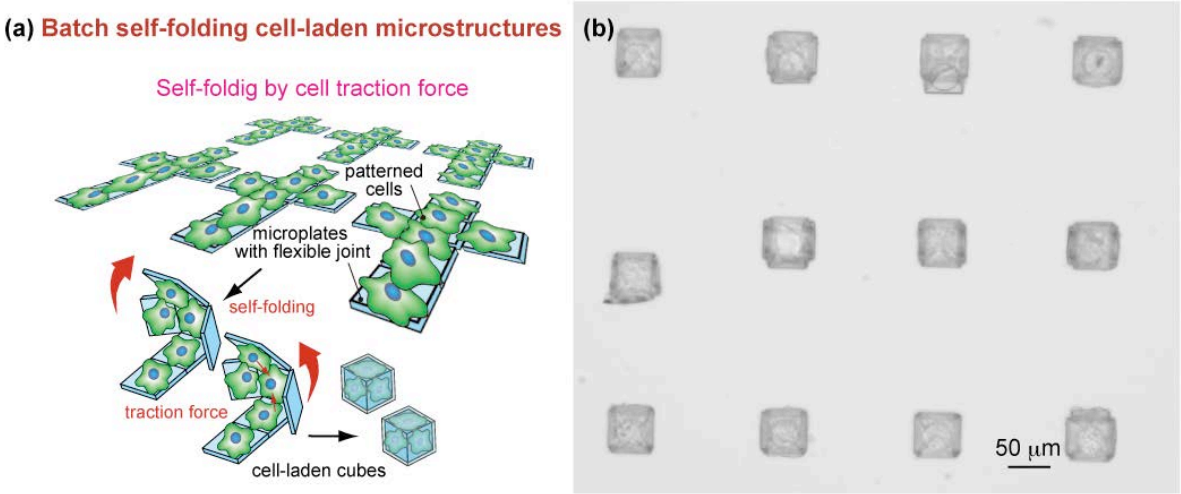

Fig. 1 (a) Conceptional illustration of batch self-folded cell-laden microstruces for 3D cell constructs. (b) 3D microstructures with concentration of 1200 structures/cm^2 were produced once.

Acknowledgment: This works have been done by a collaboration project with Prof. Shoji Takeuchi, University of Tokyo, Japan and Dr. Hiroaki Onoe, Keio University, Japan.

Origami on the iPad

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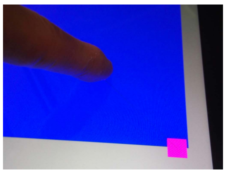
Abstract

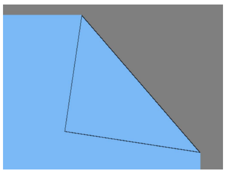
This research is to develop a system that makes Origami in iPad screen with gesture on the touch panel of iPad. We are interested in human computer interaction that investigates what gesture interface is suitable for Origami on computers. There are two problems about Origami on computers: one is how we fold a sheet of paper in computer as similar as real Origami, and the other is how we get the same shape of Origami in computer as the real.

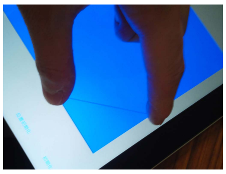
As related researches that makes Origami in computers, Miyazaki et al.[1] developed the system of making Origami interactively using mouse. Mitani and Suzuki[2] proposed the model to construct the conformation of Origami in computer graphics. Our system is a new challenge to make Origami by touch gesture insted of mouse used by other Origami system. Our goal is to fold and construct Origami just as if we fold with a real sheet of paper in the hand.

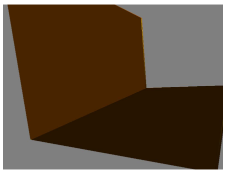
In the developed system we can use gestures using multiple fingers which are recognized by iPad. The system provides two types of gestures to make a Valley fold. The first one is that we fold a sheet of paper at a perpendicular bisector made by the movement of swiping the vertices. This gesture is similar with real Origami, but can not fold a sheet of paper in three-dimensional. The second one is that we fold a sheet of paper at the line specified by a pinch in/out gesture. This gesture is not similar with real Origami, but can fold a sheet of paper in three-dimensional because we can specify any angle of folding.

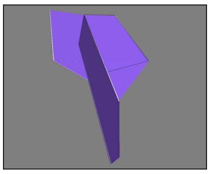
In the system we have developed previously, it was already possible to make Valley fold, Mountain fold as a reverse of Valley fold and Square basic form. In this paper, we report an implementation of another folding, that is Inside reverse fold which is a combination of one Valley fold and two Mountain folds. By making use of Inside reverse fold, we can fold not only an airplane or a rocket but also slightly more complex Origami such as a bird. In order to achieve Inside reverse fold, we have to be able to fold a sheet of paper from any angle, and a new gesture is required to specify Inside reverse fold — like a pinch out gesture.


Valley fold by long press gesture.


After long press gesture.


Valley fold by pinch inout gesture.


Perpendicularly fold after pinch inout gesture.


An airplane.

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Experimental Analysis of Self-Folding SMA-based Sheets for Origami Engineering

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Abstract

The practice of creating three-dimensional structures through the folding and manipulation of two-dimensional building-blocks has great potential in the context of engineering design. The goal of this research is to characterize the capabilities of a concept for a self-folding reconfigurable sheet for use in origami-inspired engineering design. The sheet consists of an active, self-morphing laminate that contains two shape memory alloy (SMA) mesh layers and a passive compliant medium between these layers. The SMA layers can be thermally actuated, allowing bending to occur in both positive and negative directions to create soft hill and valley folds, which differ from the sharp creases typically present in origami in two ways. First, the radius of curvature at the fold is highly dependent on several design parameters. Secondly, soft folds that occur in the structure are completely reversible, allowing the structure to fold and unfold without permanent deformation. Unlike past work on self-folding structures, these sheets can have folds along any line, be subsequently unfolded, and then be folded again in a new way. The design of the sheet allows for such bending to occur where specific regions are heated and without requiring predetermined fold locations. This work expands upon previous efforts by focusing on the experimental methods involved in the production and refinement of the structure's design. These include infrared thermographic analyses to quantify the effectiveness of embedded thermal actuators and three-dimensional digital imaging correlations to measure the effective radius of curvature along a fold line, which is a primary performance metric. Finally, we discuss the capability of the sheets to be used to create useful three dimensional structures from a combination of multiple simultaneous folds in an origami engineering context. These will include a basic cubic shape, for comparison to other active material folding methods, and simple structures requiring compound folding.

Keywords: Shape Memory Alloy, SMA, origami, active materials, self-folding, reprogrammable

Generating Folding Sequences from Crease Patterns of Flat-Foldable Origami

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The most common form to document origami pieces is through origami diagrams, which are step-by-step sequences, composed of figures that represent the state of the folded paper, in combination with lines and arrows indicating the position of the folds and the movement of the paper. With the development of modern techniques of origami design, the range of achievable shapes has increased drastically and the crease pattern (the pattern of creases left on the paper after folding an origami model) has gained importance as an efficient method of documenting origami. However, the disadvantage of crease patterns is that it is difficult to use them to re-create the design, since crease patterns show only where each crease must be made and not folding instructions. We introduce a system to create folding sequences from crease patterns of flat origami by modeling the origami steps as graph rewriting steps. By identifying the rewriting rules for an origami step, we can unfold them using the rewriting step and then simplify the origami gradually until we reach the empty crease pattern. Figure 1 exemplifies the workflow of our method. We expect that this system can help people inexperienced in folding crease patterns as well as semi-automatize the time-consuming task of drawing diagrams.

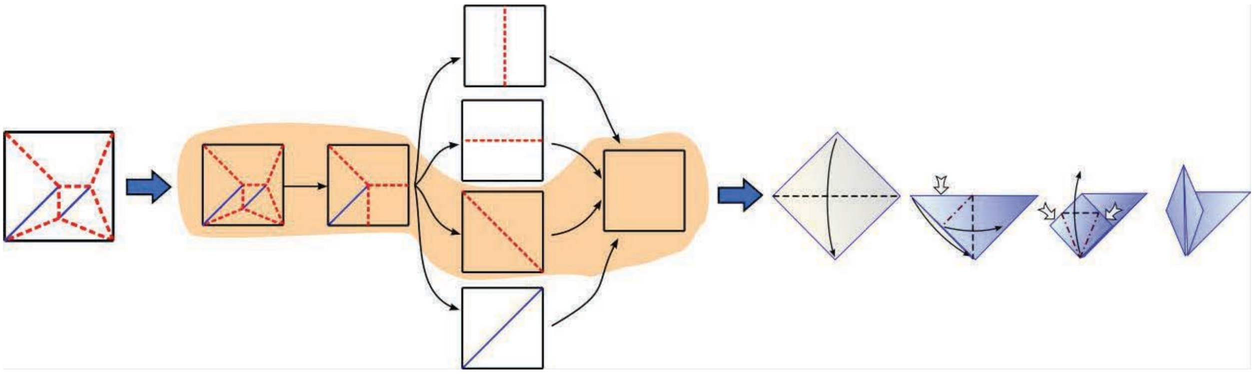

Figure 1. Proposed method's workflow. (Left) Input crease pattern; (Middle) step-sequence graph generated with successive simplifications; (Right) origami diagrams generated using the sequence highlighted in orange in step-sequence graph.

Curved Origami for Developable Surface Coupling

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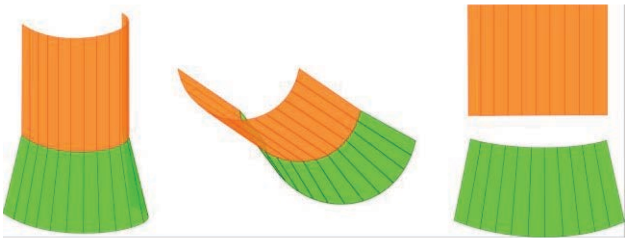
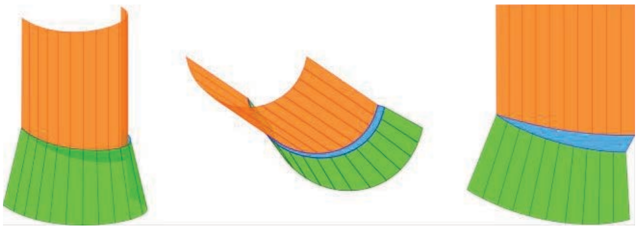
Curved origami can produce interesting curved surfaces with a reduced number of creases and is an efficient means to design 1-DOF transformable structures [1]. However, curved origami is a relatively new research topic, and it has not been investigated as thoroughly as origami constructed with straight creases only (or *prismatic origami* [2]). Specially, curved origami design does not count with much theoretical assistance as the prismatic case. Our motivation is to give the first step in creating a system similar to Origamizer [3] but using curved creases. We consider a curved origami as a collection of developable surface patches connected by curved creases, and we analyze the conditions regarding this connection, using the discretization of developable surfaces as quad meshes.

Inspired by some existing origami models, we propose a method to couple (i.e., link or unite) arbitrary developable surfaces, even if they do not obey the necessary conditions to be coupled with a single curved crease. As shown in Figures 1 and 2, this is achieved by the use of another developable surface that does not belong to the original geometry of the desired model, but is necessary to connect the two original ones while maintaining the developability of the whole structure. After the description of the method, we present a case study to exemplify it and discuss the results. We expect that this technique can be used to assist in curved origami design and help the creation of novel forms and structures based on curved origami.



(a) (b) (c)

Figure 1. Example of a cylindrical (orange) and conical (green) developable surface patches that cannot be connected by a simple curved crease. (a) Front view, (b) upper view and (c) developed form.



(a) (b) (c)

Figure 2. The surface patches shown in Figure 1 connected by a third surface (blue). (a) Front view, (b) upper view and (c) developed form.

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Rigid Folding of Periodic Triangulated Origami Tessellations

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Producing origami tessellations from sheets of thin and hard materials is an essential process for materializing applied origami structures such as fold-core sandwich structures. Folding motion of such material can be modeled as that of rigid origami, in which the bending of material or the continuous traveling of creases is forbidden.

In this paper, we investigate the continuous rigid folding motion of an origami tessellation structure whose pattern is infinitely tessellating a plane with periodic symmetry. As a study model, we analytically and numerically describe the kinematics of Waterbomb tessellations and Ron Resch's triangular origami tessellation by assuming that the fold configuration as well as the pattern is periodic. As a result of the geometric constraints, the global form of these tessellations are always restricted to cylindrical configuration unlike folding of a finite boundary-free origami tessellation allowing a double curved surface. In such triangulated patterns, the number of fold angles representing the configuration of the fundamental figure equals the number of equations from rigid origami constraints. However, this forms a 2 degrees of freedom mechanism instead of being statically determinate. One of the degrees of freedom represents the folding-unfolding motion and the other represents the change in the orientation of the rolling axis.

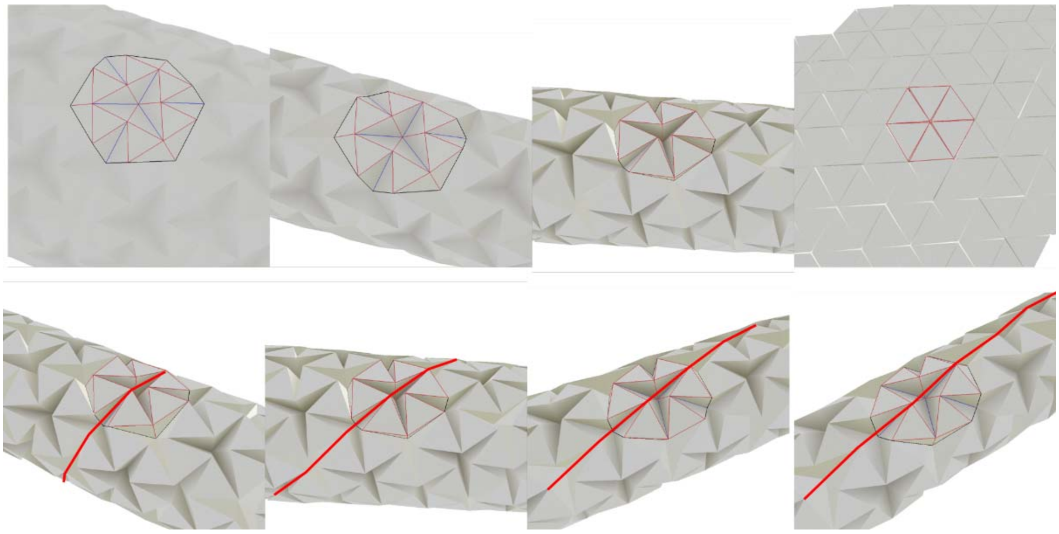

Figure: Periodic folding of Resch's pattern following a cylindrical form. Folding motion (top) and the continuous change in the rolling axis (bottom) are shown.

A New Scheme to Describe Twist-fold Tessellations

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Origami tessellations usually consist of a main design element (molecule) that is folded in a repetitive and interlinked manner to produce a tessellated design. These designs are usually folded on paper that has been prepared by first folding a square- or triangular-grid. Origami tessellations that use twist-folds often result in patterns that have a woven or weave-like appearance.

Currently, crease patterns are used to transmit knowledge of how to recreate twist-fold tessellation patterns. It can be difficult to get an intuitive feel for a folding sequence from crease patterns and describing a pattern verbally can be difficult as the terms available to refer to twist-fold units are imprecise. For example, a term like "square twist" is used when there is really an infinite variety of square twists.

A scheme is proposed herein for a precise nomenclature for the polygonal elements used in twist-fold tessellations and the interactions among these elements. The nomenclature uses size and spacing terms that are referenced to grid units and are thus valid on any grid.

The precise nomenclature makes it possible to mathematically describe relations among pattern variations. As an example of the utility of the nomenclature a 3.6.3.6 tessellation pattern is analyzed to elucidate order that can be found in its infinite variety.

Finally, the symbols provided by the nomenclature can be combined to produce objects that resemble structural formulae used in chemistry. These structural formulae contain all information necessary to reconstruct the folded pattern without the use of a crease pattern or any accompanying text. A variety of structural formulae are shown in Figure 1, with panel b showing that for a 3.6.3.6 tessellation.

Key words: tessellation; crease pattern; twist-fold

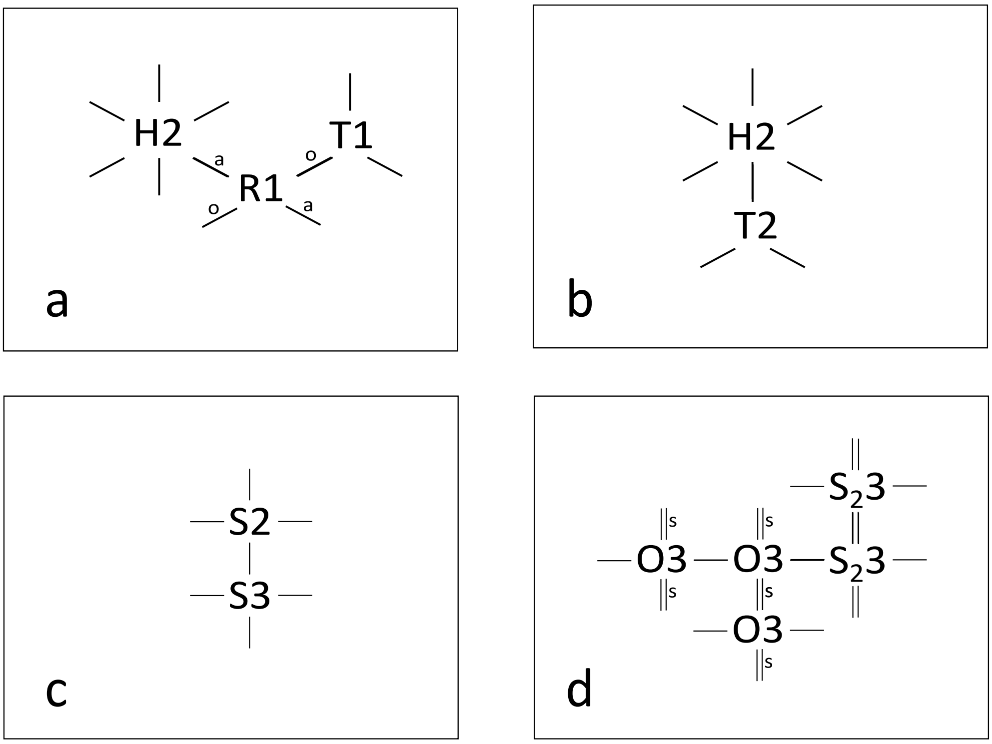

Figure 1. Sample structural formulae.

Flattening Polyhedra with Two Adjacent Rigid Faces

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Keywords: flattening, Platonic polyhedra, rigid.

Can we flatten every polyhedral piece of paper without tearing or stretching it? A *flat folding* of a polyhedron is a folding by the creases into a multilayered planar shape without self-intersection. It was proved by authors [1] that each Platonic polyhedron P can be flattened with continuous motion so that two of its faces are rigid, that is, there are no crease on them. Those two faces are parallel when P is not a regular tetrahedron, and adjacent (which means they have a common edge) when P is a regular tetrahedron. Any tetrahedron is also flattened continuously with two adjacent rigid faces ([2]). We show many examples of convex polyhedra with two adjacent rigid faces, each of which is flattened with continuous motion. Figure 1 shows a process of continuous flat folding of a regular octahedon with two adjacent rigid faces. First, two faces are flattened to comprise a bipyramid with six equilateral triangles; next, two pairs of adjacent faces are flattened where these processes are continuous.

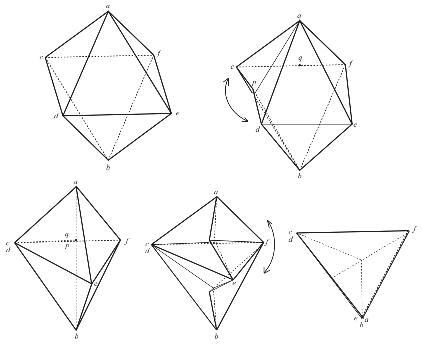

Figure 1: Continuous flattening process of a regular octahedron with two adjacent rigid faces.

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A Study on Orthogonal Pleat Tessellation and Its Folding Sequence

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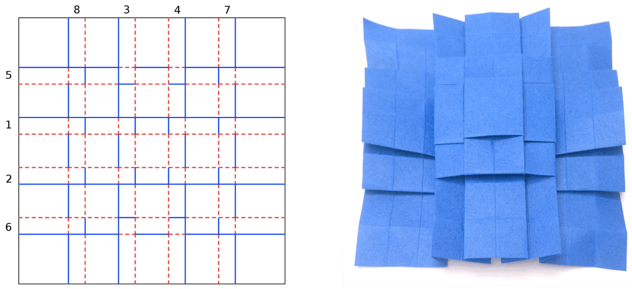
Pleat tessellation is a simple origami pattern that contains only pleat folds. A pleat fold is formed by a pair of parallel mountain fold and valley fold. Despite the simplicity, pleat tessellations have their own advantages such as the easy generation of 3D shapes. Although the definition of pleat tessellation is very straightforward, it is not easy to find the folding sequence, normally not unique, from its crease pattern which leads to difficulties to fold (Figure 1). In this paper, we study a very common form of pleat tessellations that contains only orthogonal pleats. For this particular kind of pattern, we want to simplify the universal origami theorems and to develop better algorithms for checking the foldability. We enumerate all its eight basic units which appear in crease patterns of orthogonal pleat tessellations (Figure 2) and propose a new notation to rewrite the crease pattern. Based on this notation we also notice the existence of a combination of pleat units with no standard folding sequence which is referenced here as cyclic pattern. We describe the characteristics of the cyclic pattern and propose an algorithm to detect it in its crease pattern. We believe that the cyclic pattern also has an effect on the physical properties of the origami. Finally, we provide an algorithm to compute folding sequence of an arbitrary orthogonal pleat tessellation from its crease pattern.



(a) Crease Pattern (The numbers are its folding sequence found by our system.) (b) Folded Origami (Photo)

Figure 1: An example of pleat tessellation

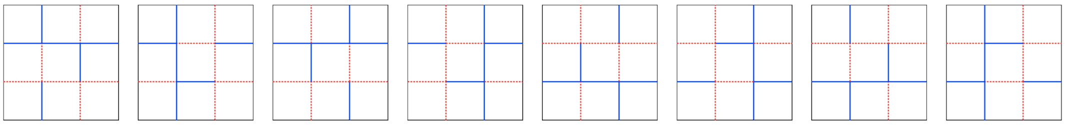
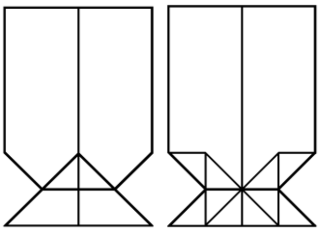


Figure 2: Eight basic pleats units

Designing with Bow-Tie Modules



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The bow-tie module may be traced back to Fröbel works in 19th century. For more than 100 years it has been recognized as finished model, sometimes joined into larger arrangements with glue. We discovered that this simple model offers firm flap-and-pocket locking mechanism what opened way to new designs.

The module is very unique. It is simple in folding and assembly, offers different ways to join modules together, works with simple joining units, bow-tie molecule may be replicated and placed along a strip of paper and finally it may join modules of different size.

Potential for origami designers and variety of models come from flexibility offered by bow-tie molecule. Placed at the end of a paper strip it creates a flap-and-pocket lock that may be used in two arrangements. It produces different mosaic designs as well as three dimensional arrangements based on polyhedral structures. Doubled molecule placed in the middle of a strip creates a flap-and-pocket joint for two crossing strips resulting in woven artwork. The molecule may join different size of modules as well as simple joining units may be used to enrich design. The molecule enables change of width of paper strip and color change. Finally, as the molecules may be located in any place of a paper strip, natural flexibility and tension of paper may be utilized in a model. Bow-ties may also cooperate with other systems of modules, especially in macro-modular approach when macro-modules utilize firm connection of bow-ties and than other system is used to assemble macro-modules into a final model.

Because of the unique features of bow-ties, they may be used in a wide range of origami activities: simple models in preliminary education including kindergartens, planar and spatial geometric models in mathematical education and finally freedom of form in art design.

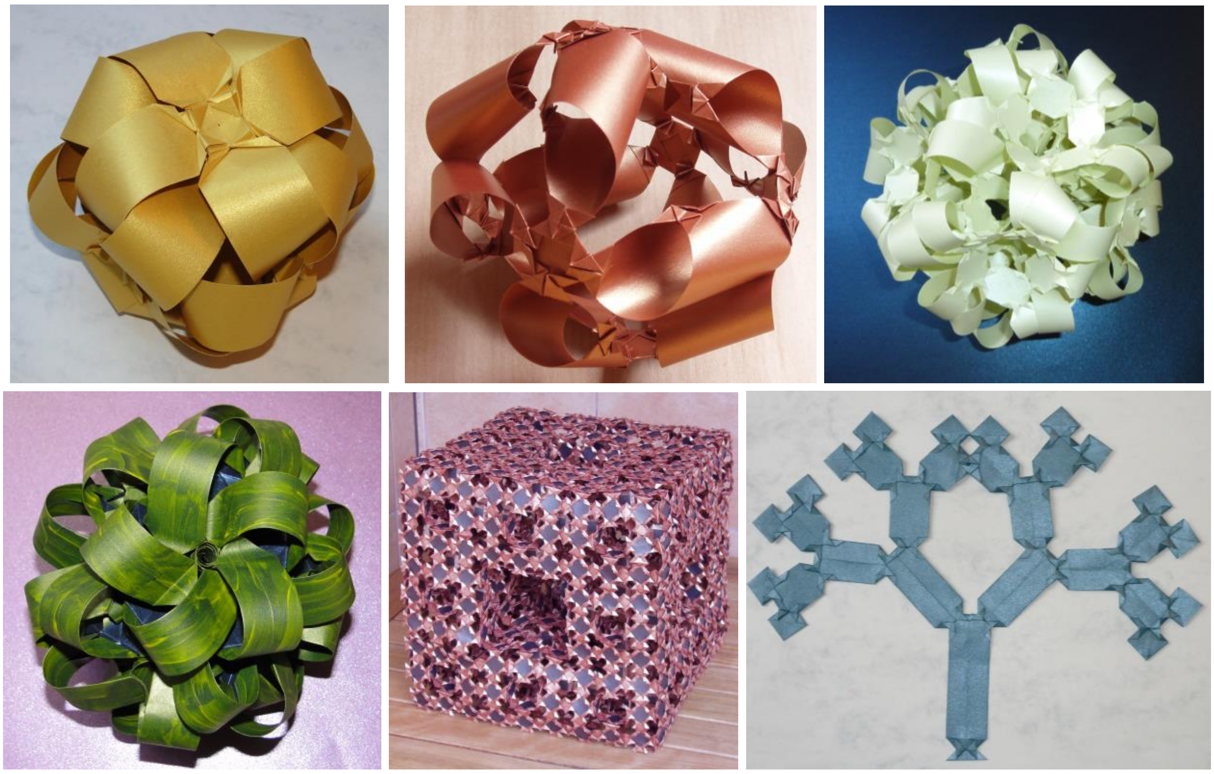

Figure 1: Examples of bow-tie models: 6-ring, tubular octahedron, Seifert surface, bow-ties combined with twirls, Menger sponge, Pythagorean tree

The Design and Production of a Folded Felt Stool

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This paper describes a design project utilizing a continuous sheet of flexible felt that is stiffened in patterned sections and folded into a piece of furniture. The strategy of creating selective rigid panels from a continuous flexible substrate may be advantageous in many applications and situations.

Designs requiring structural non-paper materials call for some combination of flexible and rigid elements or implementation of a surrogate fold that allows rigid material to become locally flexible. In this case, the wool felt sheet is initially flexible, and following a carefully developed pattern, certain areas are impregnated with resin that cures to create rigid panels. The non-impregnated areas remain flexible to serve as fold lines in the felt pattern which then can be folded into a three dimensional form. Wool felt was chosen for its aesthetic qualities as well as its non-woven properties and liquid absorption capabilities.

We discuss design iterations, crease pattern development, template creation, relevant thick-folding issues, controlling resin saturation and fold line optimization. Our folded felt stool can be shipped and stored flat and represents a novel entry in the archetypal three-legged stool tradition. We also show other furniture employing similar design and fabrication methods.

On Quadratic Curves Generated from Quasi-Fish Bases

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Orizuru is a typical example of origami. The geometric property of orizuru crease patterns has been studied in great detail. Generalization of the bird bases is closely related to quadratic curves(J. Justin 1994, Kawasaki 1998). Figure 1 shows the traditional fish base. We call the origami at left in figure 2 "*fish base* ", where four sides of the paper are on a common line. Fish bases can also be generalized in the same way as bird bases by changing creases or the shape of the paper. Figure 3 is one of deformation with the condition that the four sides are kept overlapped on a straight line. We call it a "*generalized fish base*". By loosening the condition, we can deform it as the adjacent two sides align on a straight line and another two sides align on another straight line, seen at left in figure 4. We call it a "*quasi-fish base*(QFB)". At right in figure 4 is a crease patern of a QFB. The quadrilateral for QFB must be a kite shape, seen at right in figure 4. Rotating angle ∠PAQ, we get another QFB(Fig. 5). Kawasaki thinks of the angle ∠PAQ as a ray from a lighthouse and proposed the following problem.

Lighthouse problem : An angle ∠XOY, a point A and a positive constant α $(0 < \alpha < \pi)$ are given and fixed. We take an angle ∠WAZ of α. Let P be the intersection of OX and AW, Q be the intersection of OY and AZ. And then we consider the line PQ. Find the envelope for a collection of lines PQ, when the angle ∠WAZ is rotated around the point A(Fig. 6).

We compute the envelopes and find that they are quadratic curves, and when $\alpha+\beta = \pi$, they are parabolas.

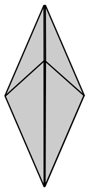

Fig.1 Traditional fish base.

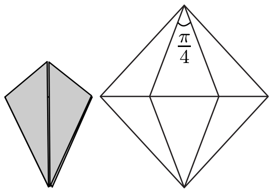

Fig.2 Fish base.

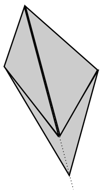

Fig.3 Generalized fish base.

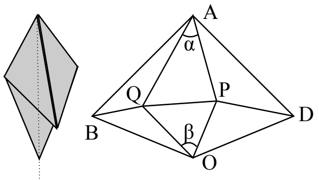

Fig.4 Quasi-fish base.

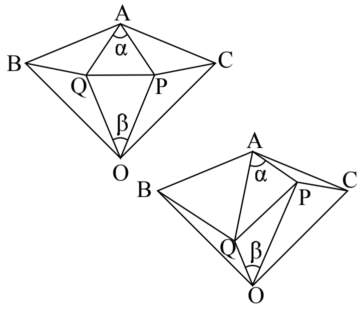

Fig.5 Two quasi-fish bases.

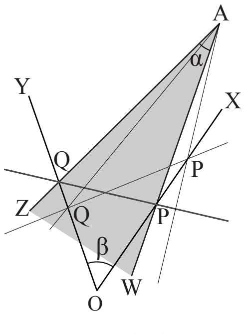

Fig.6 Lighthouse problem.

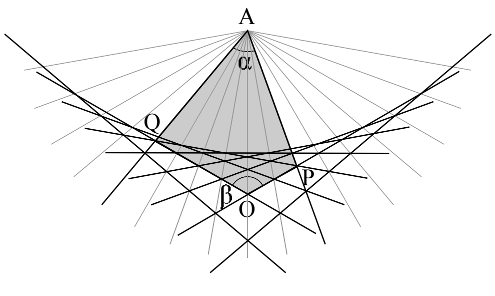

Fig.7 An example for $\alpha + \beta = \pi$.

Automated Construction and Proving of Knot Fold by Eos System

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Abstract

We present new functionalities of automated theorem proving of our computational origami system Eos and its application to the polygonal knot folds. Eos has been developed and used for over a decade to assist origamists to construct and reason about origamis by computers. The engine of Eos consists of a solver and a prover that work on the systems of multi-variate polynomials. It is implemented in Mathematica 9. The solver works in both symbolic computing and numeric computing modes. The user interface of Eos is provided by Mathematica notebook. It accepts a user specified function call in the language of Eos that triggers Huzita's basic fold operations, and outputs graphics of the origami performed by the input operation. In addition, the user interface accepts other various functions that enable the origamist to manage their origami under construction, such as unfolding, putting new points, adding automatically generated named incident points etc. The origamist performs the construction interactively and after the completion of the construction s/he can produce constructed origami repeatedly by executing the constructing procedure stored in the notebook. Furthermore, after the construction the origamist can proceed to verify the geometrical properties which s/he desired to establish by calling the theorem prover. The verification completes with the automatic generation of the *ProofDoc* that describes in detail the process of the construction and the proof.

Since the engine of Eos performs algebraic computation, we are able to combine, in a single framework, the construction and verification using common algebraic expressions. We observe that all the Huzita's basic operations are expressed logically in the formula of the form $\exists x : \mathit{Line}.\, \mathrm{O}_i(t_1, \ldots, t_{k_i})$ for $i = 1, \ldots, 6$, where $\mathrm{O}_i(t_1, \ldots, t_{k_i})$ is the predicate over geometrical objects $t_1, \ldots, t_{k_i}$, and represents Huzita's operation[1]. For example, $\mathrm{O}_5(A, l, B, x)$ states that x is a fold line that superposes point A and line l, and that passes through point B. Applying the fifth Huzita's basic operation is to prove $\mathrm{O}_5(A, l, B, x)$, i.e. finding the solution of x, and then to fold the origami along the line x. Likewise, we see that predicates $\mathrm{O}_1 \sim \mathrm{O}_4$ and O_6 are the combination of incident relations among points and lines.

In general, Eos can treat first-order formulas in prenex normal form, i.e.

$$\mathcal{Q}_1 x_1 : \tau_1 \cdots \mathcal{Q}_m x_m : \tau_m \,.\, \mathcal{P}(t_1, \ldots, t_k), \tag{1}$$

where $\mathcal{Q}_i$ is either existential or universal quantifier and $x_1, \ldots, x_m$ are variables of sort $\tau_1, \ldots, \tau_m$, respectively. A goal, i.e. the formula that we want to establish assuming the construction, is either universally quantified ($\mathcal{Q}_1 = \forall$) or quantified-free.

Knot fold of polygons requires the construction steps that are incapable by Huzita's basic fold operations since knot fold is not realizable, in general, by superposing points and lines alone. We need a more general formula (1) for the specification of knot fold operations. Once a polygonal knot, e.g. knot of a pentagon, is specified in this way, Eos constructs the polygon by knot fold, and then the prover of Eos proves the goal that the origamist specifies, e.g. its regularity, automatically. The prover engine embodies Gröbner bases algorithm, and it is invoked when the goal is specified by the conjunction of equalities and disequalities. We show the constructed (and proven) knot-fold n-gons for n=5 and 7, as illustrations of our approach by Eos. Other cases of odd n-gons are treated in the same way. We also discuss the cases of n being even.

(a) Regular pentagon $FEIHG$

(b) Regular heptagon $FEKJIHG$

Figure 1: Knot fold of regular polygons by Eos

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[1] We treat the six Huzita's basic fold operations he proposed in 1989, although one more presented later by Justin can be included for the exhaustive enumeration of basic fold operations that rely on incidence relations of point and lines.

Engineering Biological Tissues by Directed Assembly and Origami Folding

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Engineering human tissues has the potential to save lives by supplementing the supply of tissue and organ transplants and by enabling screening of new medical therapies before human testing takes place. However, there remain three key challenges that limit the broad applicability of tissue engineering. The first challenge is creating the tissue's structure in a sufficiently controlled fashion to ensure that it is biomimetic in structure and function. The second challenge lies in creating large enough quantities of tissue to be medically useful. The third challenge is determining how to create tissue rapidly with a high throughput, scalable process. This work is using origami-based techniques to form scalable, high throughput, biomimetic tissues. The "paper" is a thin sheet of biocompatible polymer called the scaffold. The different types of cells that comprise the tissue are placed in predetermined positions on the scaffold sheet with a directed assembly process. The scaffold is then origami-folded (ideally with self-folding actuators) to convert the 2D sheet into a 3D tissue structure (Figure 1). One key to designing a folded structure that replicates natural tissue is to place the cells in the right locations on the 2D sheet so that after the sheet is folded, the pattern of cells in 3D mimics natural tissue structure and enables the necessary tissue functions. The second key to replicating natural tissue in a folded structure is to create a fold pattern that replicates the natural symmetries of the tissue and ideally has a low number of degrees of freedom (Figure 2).

Both the design and the engineering aspects of the origami tissue engineering approach are presented in the context of creating liver tissue. From a design point of view, the origami layouts are chosen to minimize contact with the cell-seeded scaffold sheets (for example through the use of self-folding architectures that have few degrees of freedom) while still replicating the structure and symmetries of the target tissue (for example the hexagonal, radial blood in-flow structure of liver tissue). From an engineering point of view, self-folding (Figure 3) is used to pre-bias the fold direction of the scaffolds' "creases" to enable the practical implementation of low degree of freedom architectures for the contamination-sensitive application of tissue engineering.

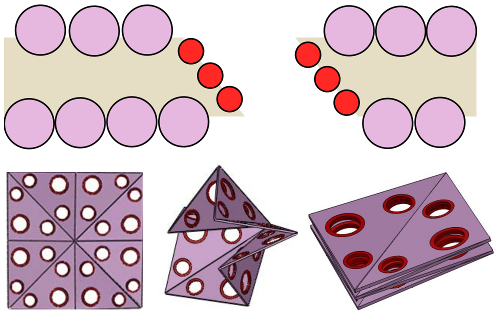

Figure 1. Above: schematic diagram of hepatic cells (pink) and endothelial cells (red) assembled on a polymer scaffold (tan) to define a path for blood flow (the through hole). Below: schematic diagram of scaffold folding to align biological structures.

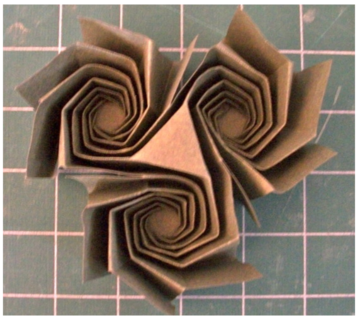

Figure 2. A low degree of freedom hexagonal fold pattern for replicating the symmetries of liver tissue.

Figure 3. Finite element analysis results of scaffold self-folding driven by bilayer polymer actuators.

A method to Fold Generalized Bird Bases

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Abstract *Orizuru* (traditional origami crane) is usually folded from a regular square of paper. Fig.1 shows a basic step of origami, orizuru, known as a "*bird base*." This geometry has been studied in great detail. K. Husimi found a method to fold the bird base from a kite-shape (Fig.2). J. Justin extended the Husimi-zuru to apply to a quadrilateral containing an inscribed circle (*QIC*) and obtained his "*perfect bird base (PBB)*" shown in Fig.3. J. Maekawa found yet another type of bird base (Fig.4). The presenter perfected the orizuru deformation theory, including Justin's and Maekawa's work, and extended it for unbounded quadrilaterals (Fig.5). Thus, the final bird base, "*Generalized Bird Base (GBB)*", was achieved.

The point J, seen in Figure 3, is named "the center" by Justin. "The center" is essential like the center of Japanese girl group "AKB48". It is determined as the intersection of two hyperbolas (Fig.6). The first passes through points A and C with foci B and D. The second passes through points B and D with foci A and C. We call this presentation "*the perfect center*" in order to distinguish it from the other "*centers*", K, that can be chosen along the two hyperbolas. Unfortunately, Justin did not show any method to fold the perfect center making it impossible to fold the PBB from a given QIC paper. In this paper, I first demonstrate that Husimi's folding method is applicable for convex QICs and is not applicable for Kawasaki-zuru for either concave or unbounded QICs. Next, I provide a method, based on generalized fish bases, to fold GBBs from an arbitrary QIC (Fig.7).

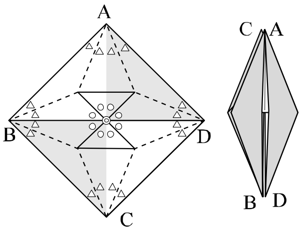

Fig. 1 Bird base

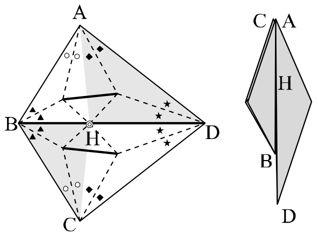

Fig. 2 Husimi-zuru

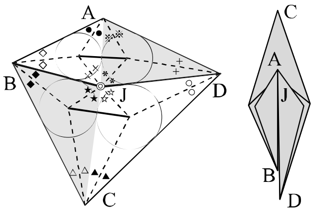

Fig. 3 Perfect bird base (PBB)

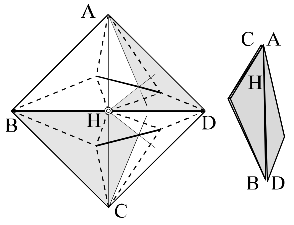

Fig. 4 Maekawa-zuru

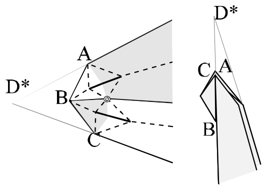

Fig. 5 Kawasaki-zuru

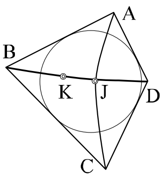

Fig. 6 QIC, the perfect center J and a chosen center K

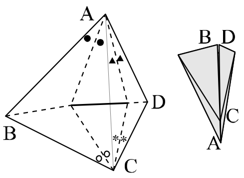

Fig. 7 Generalized fish base

A Color Symmetry Approach to the Construction of Crystallographic Flat Origami

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Keywords: Crystallographic flat origami, crystallographic groups, color symmetry

A flat origami is a type of origami in which the folded paper can be pressed in a book without crumpling. If a flat origami is invariant under a plane crystallographic group, it is referred to as a *crystallographic flat origami*. In this paper, we provide a systematic way of constructing a crease pattern with mountain and valley (mv) fold assignments that gives rise to a crystallographic flat origami using concepts in color symmetry theory. We consider crease patterns arising from Archimedean tilings using the hinged tiling method.

The method requires (1) finding the generating unit of a crease pattern, consisting of parallelograms, whose orbit under the action of a plane crystallographic group G generates the entire crease pattern; and (2) assigning distinct colors to each flat foldability type of the generating unit. By a flat foldability type of the generating unit, we mean a proper mountain and valley fold assignment will be given to the edges of the generating unit to ensure that the crease pattern folds flat. Consequently, a given assignment of the flat foldability types to the elements of the G-orbit of the generating unit corresponds to a coloring of the crease pattern. The resulting crease pattern folds into a crystallographic flat origami if the partition associated to the coloring is invariant under a plane crystallographic subgroup H of G.

For the colored crease pattern, its symmetry group $S \trianglelefteq H$ is also the symmetry group of the corresponding crease pattern with mv-assignment. The fundamental unit u of the crease pattern (the smallest region of the crease pattern which generates the pattern under the action of the translation subgroup of S) will correspond to a fundamental unit $\bar{u}$ of the folded pattern. Hence, the symmetry group of the folded pattern is also a plane crystallographic group.

This paper focuses on constructing crease patterns from Archimedean tilings with more than two flat foldability types of the generating unit. During the talk, we consider colorings involving four colors. The action of H in a coloring of the crease pattern using four colors results to the following possibilities: i) four H-orbits of colors where each H-orbit contains exactly one color; ii) three H-orbits of colors where an H-orbit contains two colors and the remaining two H-orbits both contain one color; iii) two H-orbits of colors where a) one H-orbit contains one color and the other H-orbit contains three colors and b) each H-orbit contains two colors and iv) one H-orbit of colors. By classifying the colorings into these cases, we are able to systematically arrive at the inequivalent crystallographic flat origami whose associated colored crease patterns possess certain color symmetrical properties.

To illustrate the method, consider the crease pattern $CP\ 6^3$ shown in Figure 1 (without mv-assignment), which is derived from the Archimedean tiling 6^3 consisting of regular hexagons. Its generating unit is composed of two parallelograms whose orbit under the plane crystallographic group $G = < a, x, y > \cong p6$ (a a 60° counterclockwise rotation and x, y two linearly independent translations) generates $CP\ 6^3$. A parallelogram can be given one of two possible mountain and valley assignments so that the crease pattern will give rise to a flat origami. This will give rise to four flat foldability types of the generating unit. A particular assignment of colors to each of the four flat foldability types is given in Figure 2. A bold stroke (black) denotes a mountain fold and red dotted stroke denotes a valley fold. A coloring of $CP\ 6^3$ satisfying this assignment is presented in Figure 3. There are four H-orbits of colors, where elements of the subgroup H=<x,y>$\cong p1$ of G fix the colors in the coloring. This coloring is equivalent to the crease pattern with mv-assignment in Figure 4 giving rise to the crystallographic flat origami in Figure 5.

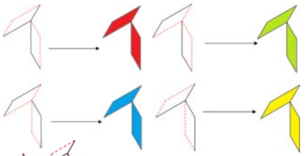

Figure 2

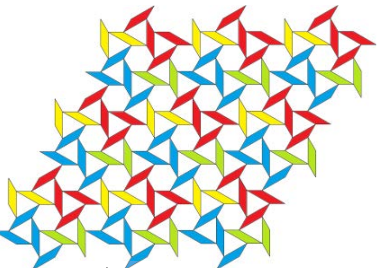

Figure 3

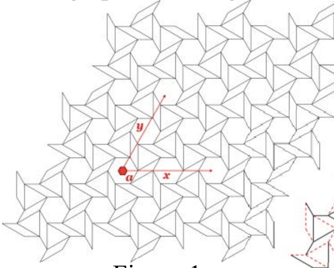

Figure 1

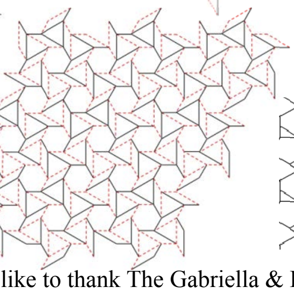

Figure 4

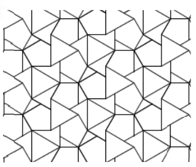

Figure 5

Acknowledgement: Taganap would like to thank The Gabriella & Paul Rosenbaum Foundation for the financial support. De Las Peñas is grateful to Ateneo de Manila University for the support under the LS Scholarly Work Grant.

Saburo Murata and His Discovery of Maekawa's and Kawasaki's Theorems

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Maekawa's and Kawasaki's theorems about flat foldability of one vertex are two of the most important theorems in Mathematics of origami. These theorems are so called because they are discovered by two leading Japanese folders/researchers Maekawa and Kawasaki in the 1980s.

It is already known that other people independently discovered these theorems, or part of them, around the same time as, or earlier than, Maekawa and Kawasaki. In his book *Project Origami*, Hull mentioned some papers involving flat foldability that were published in the 1980s and 70s in Europe.

Matsuura noticed, in 2009, that these theorems are published even earlier in two papers written in 1965 and 1966 by a Japanese mathematician Saburo Murata. Examining the contents of these papers and considering the reasons why his discovery had been buried in history until recently, we will find that the focus of the papers is not placed on flat foldability or Mathematics of origami but on art and design using paper as a foldable material.

Nishikawa's Mitate Theory and its Relevance

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Mitate or likening something to another is originally a part of the rhetoric of *haiku* poetry and can be observed in *ukiyo-e*, garden architecture, the tea ceremony, vulgar monodrama called *rakugo*, and other Japanese traditional arts and cultures. Nishikawa introduced the concept to the origami criticism in the 1990s when he characterized origami as an art of mitate. In his theory, creating an origami work is seeing geometric shapes of folded paper as something objective, such as birds or flowers.

Applying the notion of mitate to origami, however, demands careful examination because of two reasons. First, as the notion itself is multivocal and multi-faceted, we need to compare different aspects of mitate in various arts, thus clarify the meaning of mitate in origami. Only after such consideration, we can affirm, at least some extent, the relevance of Nishikawa's theory.

Second, taking into account the diversity of the art of origami, we will notice that the theory is relevant to many but not all origami models. Some origami works are intended to be realistic and do not require any mitate. Other works are abstract and involve no mitate at all. In the historical perspective, we will find that the notion of mitate is, surprisingly, more relevant to Western traditional origami than to Japanese counterpart.

The Report on an Origami Class at a Computer College

Akiko Yamanashi

From 2006 to 2011, classes on origami were given as one of the art classes in CAD department at computer college. The purpose of these classes was to improve the geometric sense such as understanding the relationship between 2 dimensional figure and 3 dimensional figure .

Generally, the students read the diagram and folded origami by themselves in class. They made origami of various categories such as animals, plants, and practical goods as well as geometric figures.

The students were bewildered first, because origami is often recognized to be child's play in Japan. But as they understood the purpose and effect of the class, they actively participated in folding origami. Although learning ability and previous experience in origami was different among individual students, they were able to correspond to their learning level since one project of origami can be easily applied to various skills.

As a result of these classes, students improved their speed and accuracy in reading the diagram in a short amount of time. The effect to improve the ability to connect the figure and actual object was recognized.

As it shows, it can be said that classes on origami are suitable for school education. It was especially effective to read the diagram by themselves and fold origami.

However, the teaching method for origami is not established at all and it is up to the experience and creativity of the teacher. In order to input origami into school education smoothly, it is necessary to establish the teaching method for origami.

Rotational Erection System (RES): Origami Extended with Cuts

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Rotational Erection System (RES) is a design method to make 3D structure from a single sheet with systematic cuts and folds. It is Origami extended with cuts or Kirigami* folded into 3D shapes. RES would be an efficient production method for both artistic and industrial use. A variety of RES design examples will be presented. RES pattern making tool implemented with a mathematical software GeoGebra will be demonstrated.

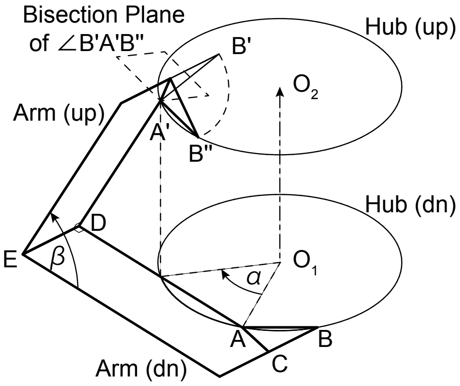
GEOMETRIC PRINCIPLE

The primary element of RES is an arm linked to a hub with a triangle tab. The arm rotates vertically around the hinge on the ground and the hub ascends with horizontal rotation following the tab linkage. The hinge line at arm-tab is defined as the intersection line of the arm plane and the bisection plane at ∠B′A′B″. The structure is bistable due to the displacement of the arm's top end outwards during the movement while there is no displacement on the ground and at the erected position.[Figure.1] General 3D shapes with a tilted hub at the top without rotational symmetry can be designed with individual arm-tab calculation.[Figure. 2]

DESIGN DEVELOPMENT

Arm profile variations generate dome and prismatic shapes. Tower shapes are made with multistage configuration.[Figure. 3]

* Kirigami: Japanese[kiri = cut + gami = paper]



$L = DA = 2\ R \sin(\alpha/2)\ /\ (\ 1 - \cos\beta\ )$

$H = O_1O_2 = L \sin\beta, \quad R = O_1A$

Figure 1. Primary Element

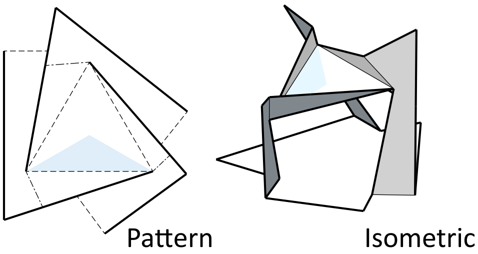


Figure 2. Tilted Hub

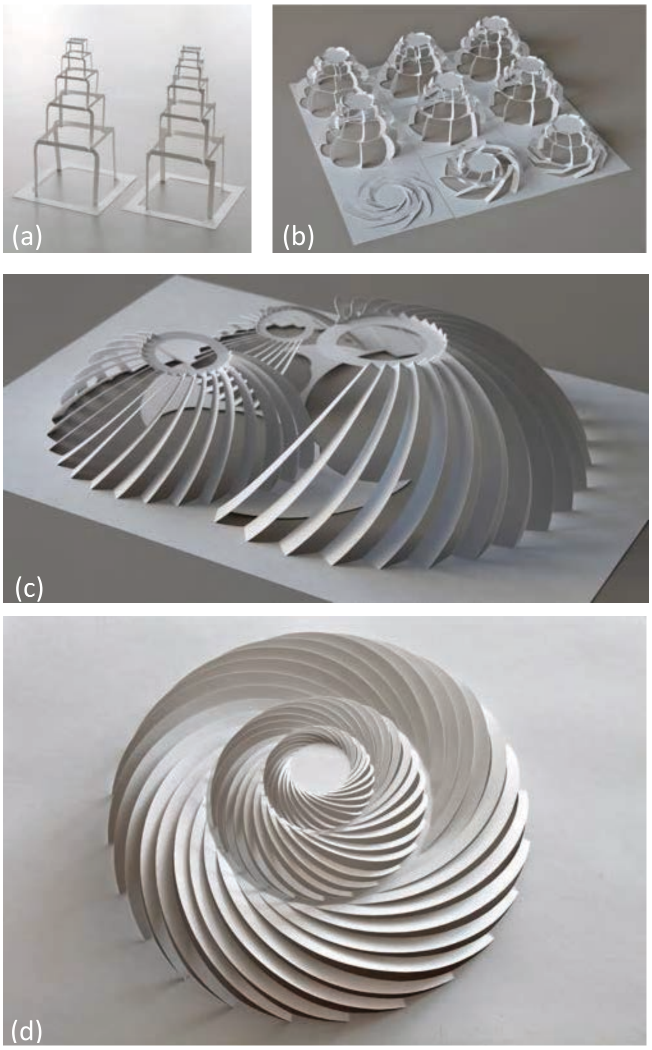


Figure 3. Towers and Domes

Miura-Ori Rectilinear Locomotion Using SMA Actuators

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Recent advances in smart materials have enabled new approaches to origami engineering, including concepts for reprogrammable self-folding sheets that contain no pre-engineered hinges or fold lines. These sheets can be programmed to fold into a 3D shape and reprogramed later to fold into a different shape. Locomotion is another capability of these sheets. In this work, we describe the use of a Miura-Ori pattern on a self-folding sheet to realize a bio-inspired locomotion technique called rectilinear motion. Often associated with snakes, this movement occurs in a straight line in which the snake scales are lifted and pulled forward, then lowered and pushed backward in a repeating pattern. Although researchers have demonstrated this pattern of locomotion in robot designs, the systems have been bulky and complicated. The use of a reprogrammable self-folding sheet is a simpler and more compact solution. The Miura-Ori fold is advantageous because of its flexibility of movement. Movement is achieved using thermally-activated Shape Memory Alloy (SMA) to provide actuation throughout the self-folding sheet. Heating patterns define localized actuation within the sheet and produce a desired origami pattern. Rectilinear locomotion can be achieved by cycling through a Miura-Ori pattern sequence. Locomotion was demonstrated by analyzing single and multiple Miura-Ori units as shown in Figure 1. The locomotion ideas are investigated using physics-based finite element analysis. The results show that the snake-like motion of the Miuri-Ori SMA design has potential for locomotion in reprogrammable self-folding sheets.

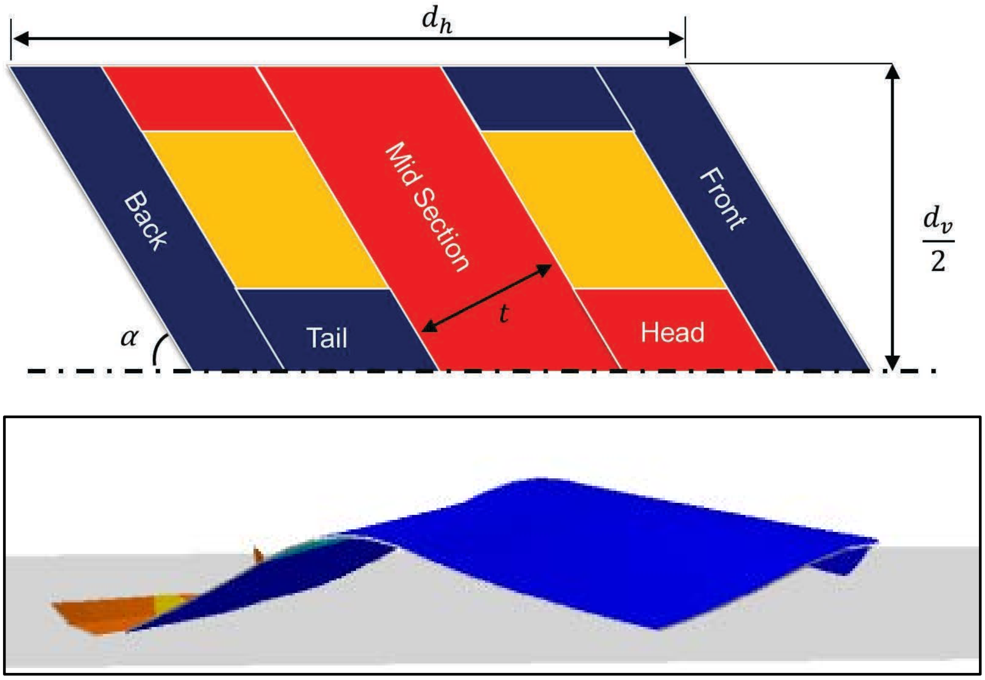

Figure 1: Top: Analysis unit with dashed line indicating line of symmetry, red areas indicating mountain folds, and blue regions indication valley folds. Bottom: Miura-Ori unit performing rectilinear locomotion.

Cosmological Origami: Folding up the Dark-Matter Sheet into the Cosmic Web

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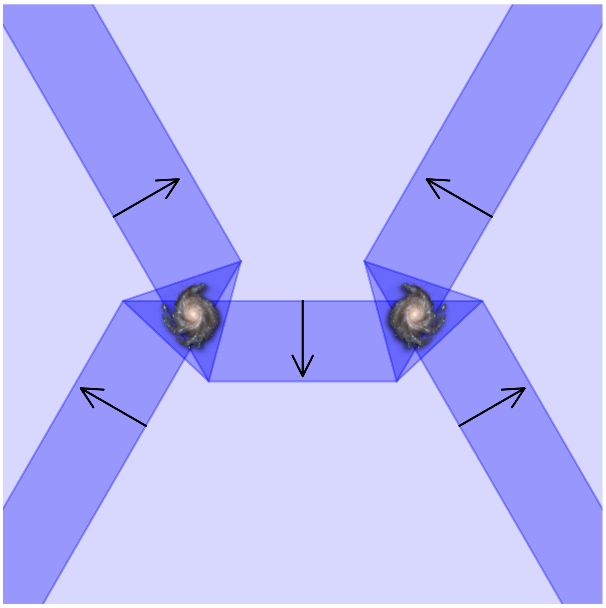
An origami analogy helps to understand the formation of galaxies, and their spatial arrangement, known as the cosmic web. In paper origami, a non-stretchy two-dimensional manifold (paper) is folded in three dimensions to build objects. In cosmological origami, an initially flat three-dimensional manifold that pervades space (the 'dark-matter sheet') is folded up to build galaxies and the cosmic web of filaments between them. This folding occurs in six-dimensional phase space, consisting of three usual position dimensions, and three dimensions of velocity. A particle with a given position and velocity occupies a point in 6D phase space. The sheet can never cross itself, or tear, just as in 2D paper origami. The creases, or *caustics*, as they are called in cosmology, are physically important because they mark the edges of structures like galaxies, and filaments of matter and galaxies. Going from paper to cosmological origami, there are two major differences: the dimensionality increases, and the sheet is allowed to stretch. Some mathematical constraints are known about the types of caustics that can form (Arnold et al. 1982), but these are very local, and origami mathematics may help to constrain global properties.

Here, I explore properties of the cosmological origami model in a 2D universe, mentioning a 3D case as well. Some patterns emerge that origami practitioners will know as origami tessellations (Gjerde 2008). The model imposes extra constraints on origami from one side, and on cosmological structure formation from the other. The origami folds flat, i.e. folding of the 2D sheet is allowed in 4D, but the result is constrained to 2D position space. Also, single-layer regions, or *voids*, are constrained not to undergo rotation from the initial state.

A folded-up example of two galaxies and the web around them is shown below. The origami term for a cosmological-origami galaxy is a twist fold. In this model, whenever a galaxy forms without stretching the sheet, it must have some rotation, and must form filaments (the thick bands) radiating from it. This helps to explain the ubiquity of galaxy spin and filaments in the Universe. I also show some properties of 'galaxies' in a 3D universe.



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Geometry controlled catastrophe and the link between snap buckling, origami, and material science

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Man-made structures are generally designed to avoid catastrophic failure, the term used to refer to the sudden loss of mechanical stability as a load or material parameter is varied. Nature, on the other hand, uses snapping and bistability for a variety of purposes in plant and animal life. In order to understand how one can create structures that switch very quickly from one state to the other, but retain mechanical stability, we turn to ideas from origami and differential geometry. We start by considering isometric deformations on discrete developable surfaces *via* the canonical Miura-ori origami pattern (see Fig. 1A-D). Using a combination of theory and experiment, we identify the origins of bi-stability in this folded structure and discuss the implications for constructing mechanical metamaterials by tessellating the unit cell. Generalizing this phenomenon to non-Euclidean surfaces, we derive a design rule that is subsequently used to predict and control swift snapping motion in a variety of thin materials. This methodology can be used to design and fabricate structures that have fully three-dimensional, mechanically stable conformations, but still remain rigid (Fig 1F).

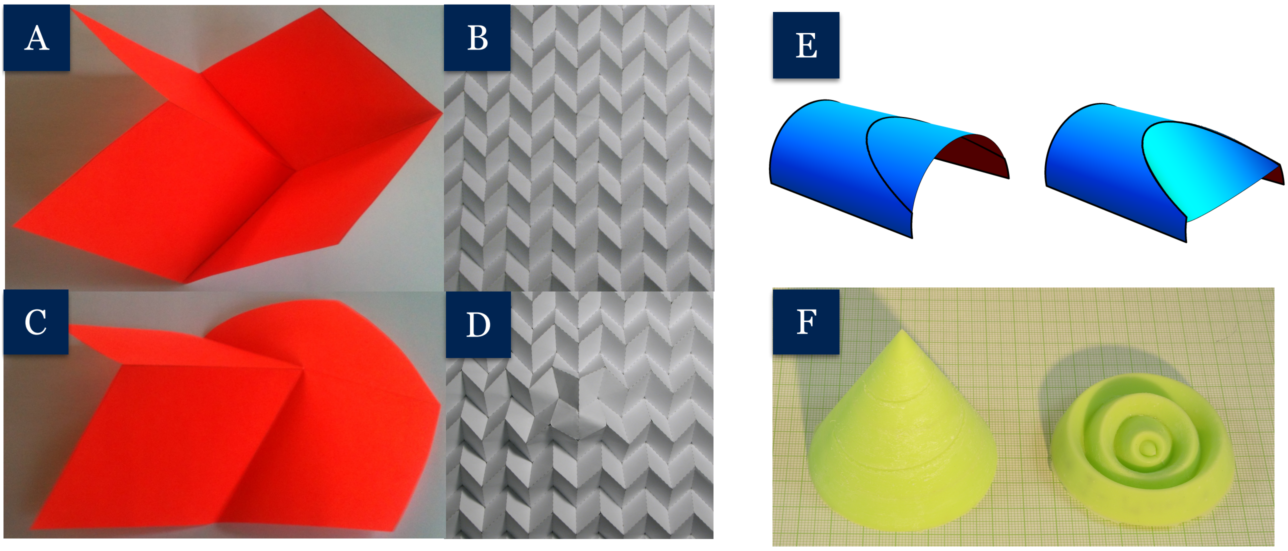

FIG. 1: A) Miura-ori unit cell in the naturally folded state and B) the tessellated form of the Miura-ori pattern. C) A secondary state is also mechanically stable, if the faces are allowed to bend. D) The introduction of one of these secondary states into the tessellated pattern modifies the whole structure and has implications for the bulk material properties. E) Non-flat structures have an analogous snapping transition that allows for discontinuous motion from an unfolded state to the folded state. F) With strategic placement of folds simple geometric figures may be swiftly and reversibly transformed to more complicated sculptures.

Designing N^{th} Order Rotational Symmetry Origami From 4^{th} Order Symmetric Crease Patterns

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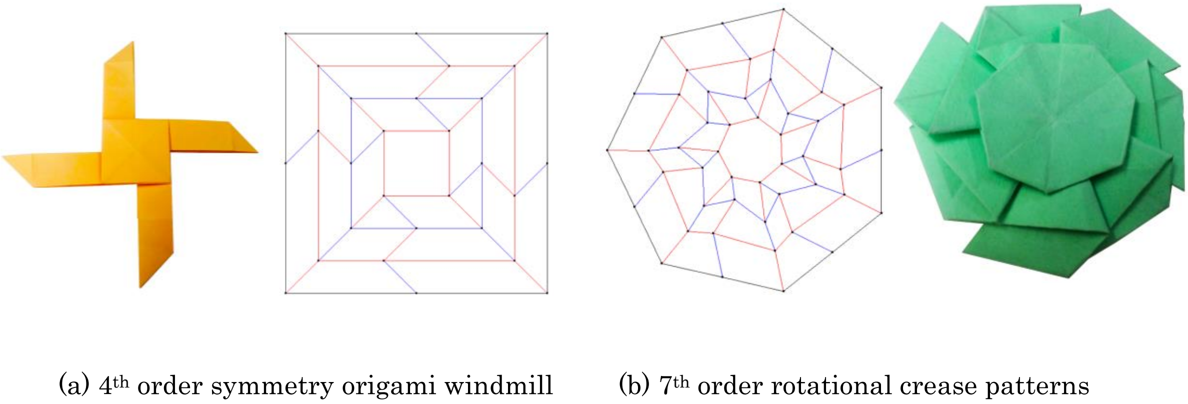
In general, origami designs begin with a square sheet of paper and, consequently, many of them have line symmetry or 4th order rotational symmetry. In this work, we develop an application to design flat-foldable origami with a paper shaped as an arbitrary regular polygon, apart from a square, in order to create models with more complex symmetries. The application can determine whether the inputted crease pattern can be folded into a flat 4th order rotational symmetrical origami, as shown in Figure 1(a). Also, it scales one sector by a constant factor of 4/n and rotates it around the center of the former crease pattern by an angle of 360/n degrees for n-1 times. Then an angle adjustment will be operated. As a result, a new rotational symmetry of order n is created, as shown in Figure 1(b).

According to Kawasaki's theorem and Maekawa's theorem, we adjust the angles of creases by rotating them around interior vertices, in order to create a flat-foldable result. The crease pattern can be finally generated by this fine adjustment. Crease patterns which referenced as radiation pattern, such as Bird Base, Frog Base, are successfully transformed. However, in some cases the new pattern may only be flat folded locally. In such cases, the foldabitity of crease patterns needs to be checked.

The system can also display the folded form of the resultant crease pattern. We expect that this tool can be useful for the creation of new origami designs.



(a) 4th order symmetry origami windmill (b) 7th order rotational crease patterns

Figure 1: An example of windmill

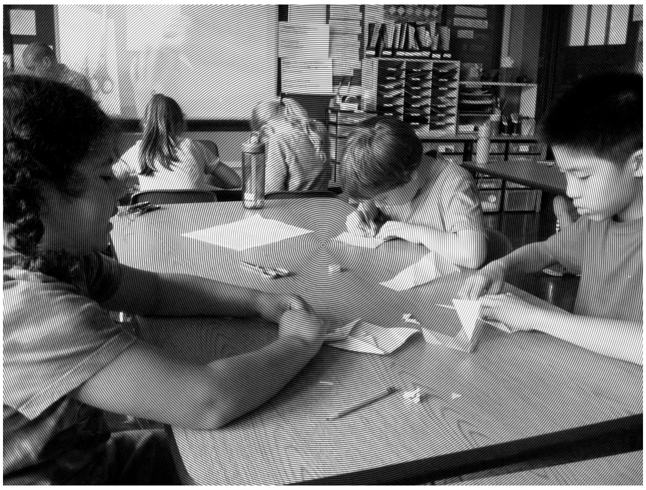
Hypothesis and Model for Evaluating an Elementary School Origami Program

Robert Orndorff Debby Halperin (Seattle Public Schools)

A small number of studies have looked for measurable properties or results pertaining to various aspects of paper folding and math achievement in schools, including Arslan; Bart (and other studies on the same topic by the same team); Boakes 2009 (and other studies on the same topic by the same scholar); Boakes 2011; Cakmak; Carter; and Wilson.

A very large number of studies are not mainly or at all about paper folding but must be accounted for, e.g., Sowell who finds that programs with concrete instructional materials, including paper folding, that last a school year or longer produce significant improvements in mathematical achievement for students of all ages, whereas shorter term programs did not result in measurable improvements.

We started our Seattle public school paper folding program some eight years ago with the aim of addressing four aspects of early science, math and engineering education, namely, student ability, interest, confidence and plans.



Over time we have added a number of other aims, e.g., looking at things deeply; failing happily and constructively; asking one's own questions; using strategies for creativity.

By some measures, our program is now perhaps the largest such paper folding program in the United States. Each year, we work with three elementary school classes. That is, each class does paper folding once per week for an academic year, or approximately 35 meetings.

According to unscientific measures, the program has been a success. For example, it is popular with teachers, students and parents. One of our teachers was a finalist for the national presidential award in mathematics teaching. On the other hand, we have not yet formally evaluated the program.

In this talk, first we will present our principles, processes and aims. Second, we will discuss our ideas pertaining to the construction of a credible story (i.e., a testable hypothesis) that could serve as the basis for evaluation and improvement. Third, we will explicitly allocate time for discussion, since we believe this will be a rare opportunity for the exchange of ideas.

Our conjecture is thus: Paper folding (with its own particular set of characteristics) done once per week for one academic year, according to a given set of principles, with carefully selected projects, by a math and origami expert, for a teacher and families who are aware of the connection between math and paper folding, will contribute to a given set of aims (as above).

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Approach for Unfolding 3D Meshes for Crafting Paper Models

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In this work we introduce a method for the automatic conversion of the 3D mesh models into a set of the 2D patterns for producing physical copy-models. The basic workflow is illustrated in Fig.1. There are also some related work for producing the paper models based on different approaches [1][2][3]. But the related researches based on a similarity of the original model, which have drawbacks such as losing details of the original model.

Key words: surface mesh, segmentation, unfolding models, mapping, pattern-net

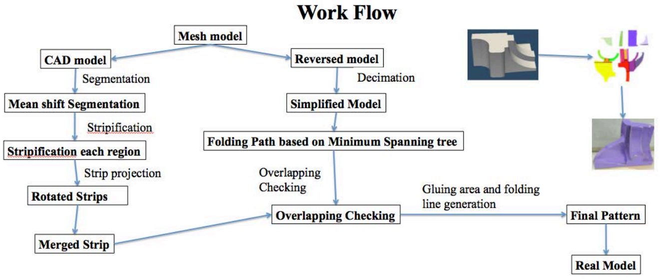

Fig. 1. Work flow of the proposed approach

In the proposed approach, the models that can be segmented into near-developable parts are decomposed into the 3D polygonal segments, which correspond to the 2D patterns. A segmentation algorithm that enables the creation of the copy-models is suggested in this paper. We use clustering technique for grouping large data into meaningful groups, or subsets. Our approach is based on clustering the normals of the triangles in a mesh. The number of clusters is minimized by merging clusters by using cluster analysis method and topological connection approach. Each cluster is associated with a cluster of triangles that is partitioned into the mesh segments by using a region growing technique.

Triangle strips are generated in the mesh segments and mapped onto the 2D print-planes for creating patterns. Mapping strips are done in two steps: 1) mapping onto the fixed plane in 3D; 2) mapping the flattened strips onto a print plane. The resulting strips are assembling in the flattened patches, or patterns, located in a common plane.

For the complicated models with very sparse mesh and when a high accuracy and stability are not necessary conditions we use a graph-based approach [4]. The method is based on unfolding the decimated model into plane by cutting it along edges according graph and dual graph. A spanning tree of the dual graph defines an unfolding of the mesh. Mesh can be unfolded into the plane for the construction of the 2D patterns by cutting all edges that do not have dual in the spanning tree. A paper copy-model is produced by gluing the printed patterns. The paper presents a set of the developed algorithms which allow us to achieve the desired goal.

To demonstrate the practicality of the proposed method we present several mesh models and their copy-models result in Fig.2.

(a) (b) (c)

Fig. 2. Result of the proposed approach. (a) Segmentation approach; (b, c) Graph-based approach.

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Geometric and arithmetic relations concerning origami

Jordi Guàrdia and Eulàlia Tramuns

Geometric instruments can be viewed as objects to construct points and curves in the plane. They also have an arithmetical approach, when we identify the plane with the complex plane, and look for the set of constructible numbers with them. But these two aspects are usually considered together, and we talk in a general way about constructions with a given instrument.

As an example, it is known that the trisection of an angle is possible with origami. And it is also known that the set of constructible numbers with origami is the smallest subfield of $\mathbb{C}$ closed under square roots, cubics roots and complex conjugation, using the set $\{0, 1\}$ as an initial set.

The first result describes essentially a purely geometric construction of a certain line while the second result refers to the algebraic structure of the origami constructible numbers and depends on the choice of an initial reference.

We present a new formalization of geometric instruments that considers these two situations separately. We introduce the concept of *tool*, which formalizes a physical instrument as a set of *axioms* representing its geometric capabilities. We then define a *map* as a tool together with a set of points and curves that will be the initial reference. We rewrite known results concerning origami using this new approach and give new relations between origami and other instruments, some obtained considering them as tools and others considering them as maps.

Keywords: tool, map, geometric construction, origami construction.

References: Ph.D. thesis *A formalization of geometric constructions*, E. Tramuns, Universitat Politècnica de Catalunya, 2012.
http://www.tdx.cat/handle/10803/101399

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Oricreate: modeling framework for design and manufacturing of folded plate structures

R. Chudoba, J. van der Woerd, J. Hegger

In order to effectively support the design and manufacturing of origami inspired structures with load-carrying function, we have formulated and implemented a general modeling framework named **oricreate** that can be used for three types of applications:

1. form-finding, including adaptation/optimization of a crease pattern to achieve desired shapes and to improve the load carrying behavior,
2. simulation of the folding process including the folding devices as the support for the design of manufacturing/folding tools,
3. structural analysis and characterization of mechanical properties of the folded structure.

The extensible optimization framework can incorporate various types of optimality criteria and equality as well as inequality constraints as plug-ins. A simple example of a design-based optimality criteria is the minimum distance of selected crease nodes to a target surface. Another example is provided by the minimum potential energy of gravity. Examples of equality constraints are (i) rigidity of the faces or (ii) flat foldability and/or (iii) developability of the crease pattern. For structured crease patterns, factory methods have been implemented in order to generate the initial planar configuration. Currently, the Yoshimura (exemplified in Fig. 1), Waterbomb and Miura-Ori crease patterns are provided.

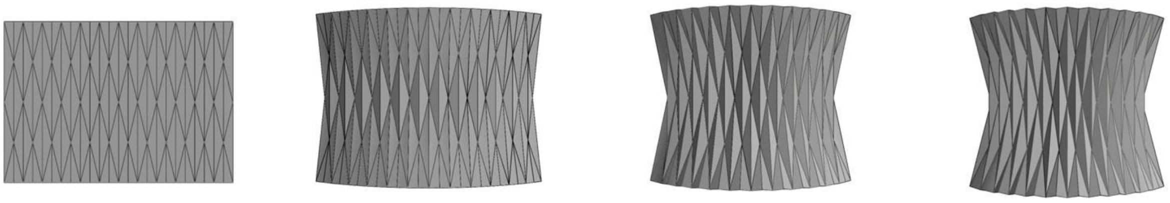

Fig 1: Example of a Yoshimura crease pattern adapted to fit a given target surface

The **oricreate** toolkit has been developed for the design and manufacturing of **oricrete** structures: thin concrete plates made of fine-grained cementitious matrix continuously reinforced in the middle of the cross section with high-performance textile fabrics (carbon or alkali resistant glass). Crease lines are introduced into the plate using linear spacers below and above the reinforcement layer. The hardened plate is folded into a desired configuration using devices exploiting the kinematics of the rigid-origami crease pattern. In the target configuration, the crease lines are fixed using mortar grout.

The paper will present the mathematical formulation of the optimization problem and present examples of simulated and realized **oricrete** elements.

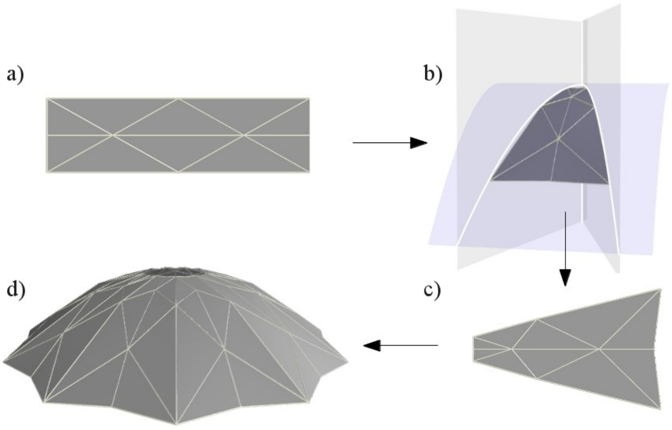

Fig 2: oridome: a) initial Yoshimura crease pattern; b) fitting to target surface; c) adapted crease pattern in the developed state; d) dome assembled from folded segments.

A Systematic Overview of Origami-Based Structures in Technical Applications

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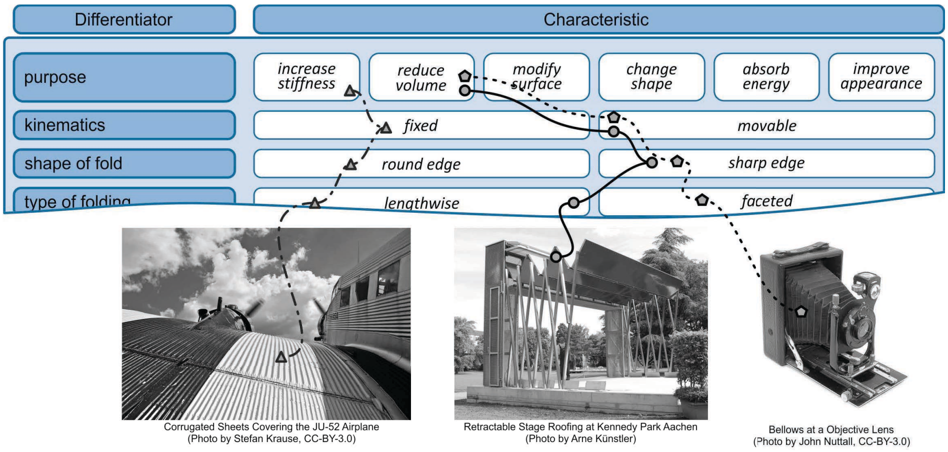
Keywords: folded structures, foldable structures, classification

Diverse types of foldings fulfill various tasks within a wide range of technical applications. Besides the most obvious subdivision of foldings into folded but immovable structures and foldable mechanisms they differ for example in the scale of the structure, in the characteristics of the folded material or in whether and how they are actuated. The high variety of foldings leads to very inconsistent descriptions of their characteristics and attributes. The demand for a higher understanding of folding only can be satisfied by the aid of a systematical overview and by a possibility to compare structures. For this purpose a consistent description method containing a suitable set of differentiators is required. Thereby it has to be considered that some differentiators may be valid for all types of folded and foldable structures, but subgroups of foldings need specific ones. Hence besides the consideration of foldings in general a special focus is on Origami-based structures.

A convenient display method is the use of morphological boxes. Figure 1 gives an outlook on such a morphological box containing a set of differentiators and their corresponding characteristics. In this well arranged scheme a folding is described by lines connecting its characteristics as shown by the chosen examples.



Corrugated Sheets Covering the JU-52 Airplane
(Photo by Stefan Krause, CC-BY-3.0)

Retractable Stage Roofing at Kennedy Park Aachen
(Photo by Arne Künstler)

Bellows at a Objective Lens
(Photo by John Nuttall, CC-BY-3.0)

Fig. 1: Different Foldings in a Morphological Box

Relocations of the completed path correlate to new foldings which may represent alternative solutions for a given problem. Comparisons of paths may reveal new applications to a given type of folding. In this way the classification and the description by the use of morphological boxes are intended to contribute to an increased use of foldings in technical applications.

Filling a Hole in a Crease Pattern: Isometric Mapping of a Polygon given a Folding of its Boundary

Erik D. Demaine Jason S. Ku

Suppose we are given a flat polygonal sheet of paper P and an isometric piecewise-linear folding B' of its boundary B. We want to find an isometric piecewise-linear mapping $f : P \to \mathbb{R}^3$ such that $f(B) = B'$ i.e. find a crease pattern that can fold the boundary as desired. Does a solution always exist?

This problem was posed (in a more specific form) by Barry Hayes at 3OSME in 2001. Its solution has many applications to modern algorithmic origami design. It generalizes the molecule problem where the mapping of the boundary is restricted to be collinear and the mapping of the paper must also lie in the plane. But the general problem could also be used to design tessellations, color-changes, and non-flat foldings.

We propose an algorithm for finding an isometric mapping for flat foldings that runs in polynomial time and argue its correctness. We define the necessary conditions for a solution to exist and analyze the solution space that the algorithm produces. Note that we do not address self-intersection in the computed folding i.e. we do not guarantee the existence of a valid layer ordering.

Acknowledgments. We thank Barry Hayes, Robert Lang, Tomohiro Tachi for helpful discussions.

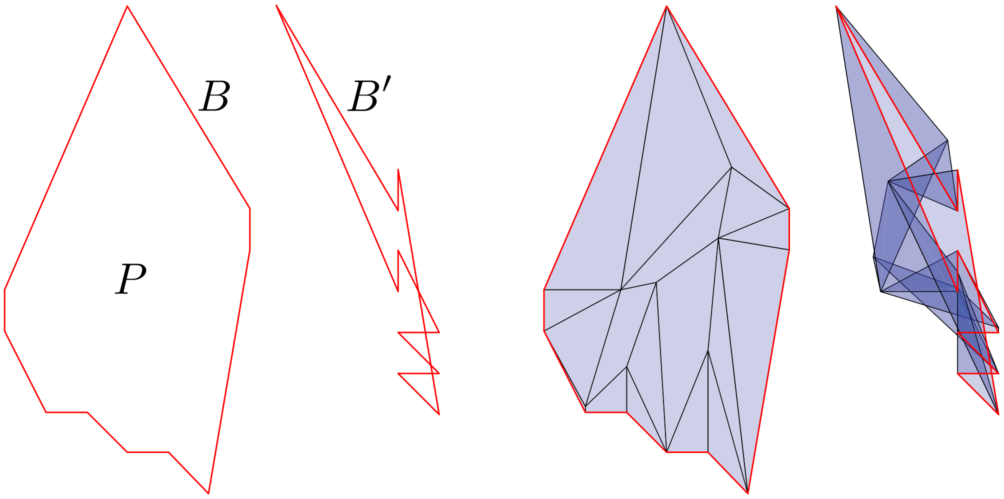


Figure 1: The input (left) is a polygon P and a folding B' of its boundary B. The output (right) is an isometric piecewise-linear mapping $f : P \to \mathbb{R}^2$ such that $f(B) = B'$.

Scaling a Surface down to Any Fraction by Twist Folding

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MIT

Martin L. Demaine
MIT

Kayhan F. Qaiser
McGill University

In this paper we show that any acutely triangulated surface can be folded down to any desired fraction of its size while preserving angles and relative lengths. Thus the folded form is simply a scaled version of the original surface. In particular, it is known that any polygon can be acutely triangulated, and we conjecture that any polyhedral surface can be acutely triangulated, making this construction extremely general.

We present an algorithm to generate the crease pattern and folded state for a given acute triangulation and a given scale factor between 1/3 and 1. By repeatedly applying this algorithm, we can achieve any desired scale factor less than 1. The algorithm divides each acute triangle into three quadrilaterals (meeting at the circumcenter), specifies the folding for each quadrilateral unit separately, and then merges the units together. We argue that there are no inconsistencies between units during the merging. Each quadrilateral folding resembles a generalized *twist fold*, popular in origami tessellations and flowers.

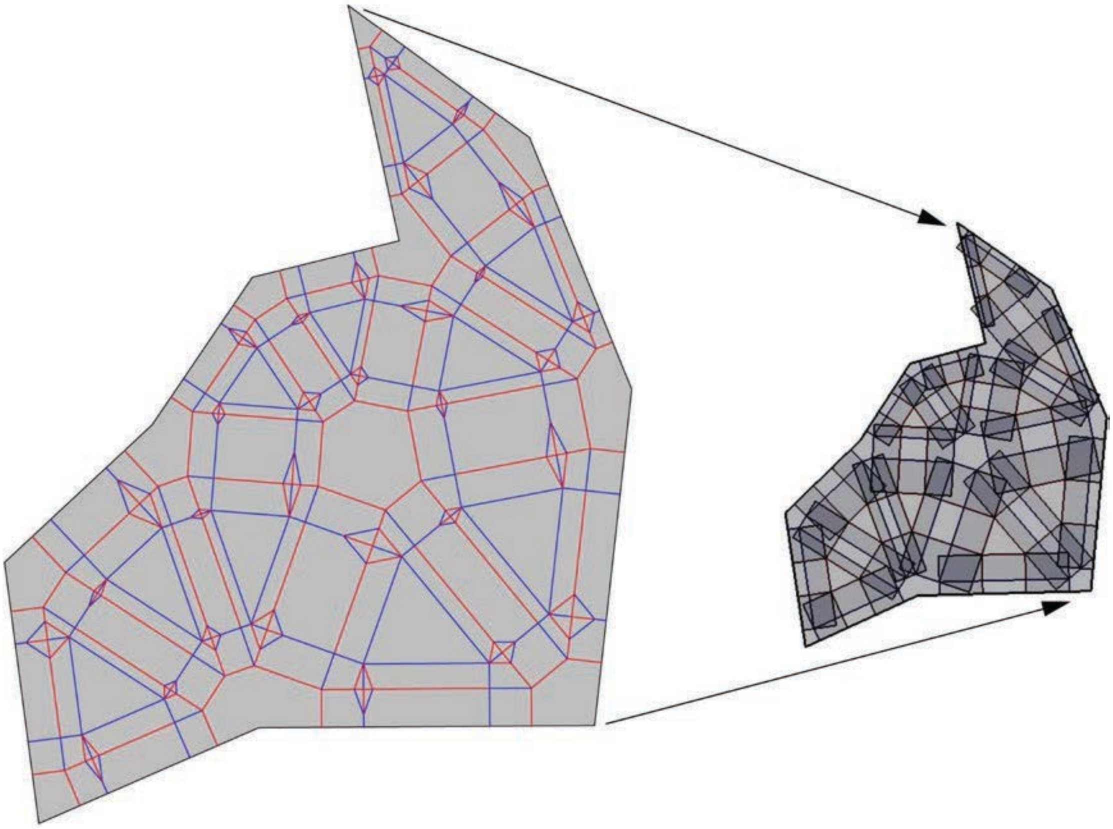

Figure 1: Dog head creased and folded to half its size. (Final diagram shown as x-ray view using Tomohiro Tachi's Freeform Origami software.)

Two Calculations for Geodesic Modular works

Miyuki Kawamura
Independent

Geodesic sphere is made by great circles on the surface of a sphere. The great circles are divided by each other and the surface of the sphere becomes to triangles or another polygons network. Some kinds of geodesic sphere can be made by useing a technique of modular origami. I designed a modular work "Cosmosphere"(Fig.1) in 1996. This model is made from 1890 modules. We need modules of three kinds of different lengths to make a ball shape. My another model "Geosphere"(2006, Fig.2) is also made from 1890 modules, but it looks like a icosahedron, not ball shape, because all of the modules for Geosphere have same length. The triangle network on the surface of the model is almost regular triangles.

To calculate how many modules are needed to make a Geosphere model, we use a 2D oblique coordinate system the angle is 30 degrees. The origin is one of the five-fold symmetry vertex, and a coordinate (m, n) is the position of the next five-fold symmetry vertex. The shape of the model is a regular icosahedron approximately, so surface area of the model and number of edges of the regular triangle network can be calculated easily. The number of the edges is given by 30(m^2 + mn + n^2), and this is the number of modules needed to make a Geosphere model. In general this equation can be used to make a geodesic sphere model, not only for the modular works.

We also calculate the number of edges for each stage when a sphere is cut into round slices. Using these numbers, we can design color combinations for a geodesic sphere model.

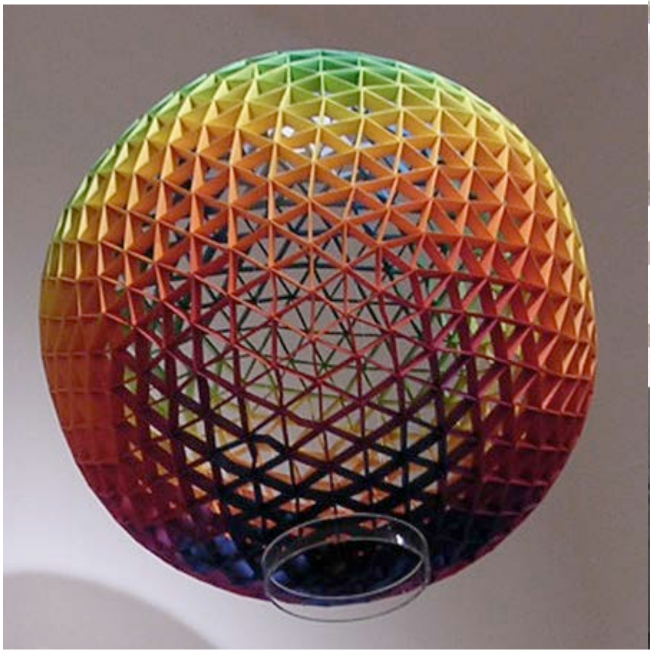

Fig. 1 Cosmosphere (1996)

Fig. 2 Geosphere (2006)

Comparison of Mechanical Properties of Periodic Non-flat Tessellations

Yves Klett and Marc Grzeschik and Peter Middendorf
Institute of Aircraft Design, University of Stuttgart

Three-dimensional tessellations have been the focus of recent research concerned with the development of new core structures for application in sandwich constructions. Several well-known tessellations, especially miura-ori (and derivatives), have shown promise as high-performance lightweight materials. The potential to integrate additional functionality into until now purely passive structural components together with total customizability of the unit cell geometry opens up new possibilities in sandwich construction.

In this context, several in-detail studies and screenings of the mechanical performance of miura-ori and related cell geometries have been conducted with the goal to evaluate, predict and optimize their performance. Because of the huge amount of possible parameter combinations even for the simplest cells, and because simple cells are usually better suited for large-scale manufacturing processes, only a tiny subset of the huge variety of potentially suitable tessellations has been studied in-depth.

We will select a number of well-known miura-type tessellations together with other, not yet tested geometries including Resch-type and curved tessellations and compare their mechanical properties with a focus on out-of-plane compression in a sandwich context (Figure 1). Samples will be prepared from one base material in a reproducible fashion with comparable characteristic dimensions and will be tested under identical conditions.

Results on compression strength, modulus and crushing behavior will be presented and analyzed together with a preliminary assessment of further properties that may be relevant for future applications. The goal of this study is to generate data for a broader variety of unit cell types and to do so in a comparable process that can be re-used and adapted for further research.

Keywords: Origami, Tessellation, Mechanical Engineering, Lightweight Structures, Sandwich Cores

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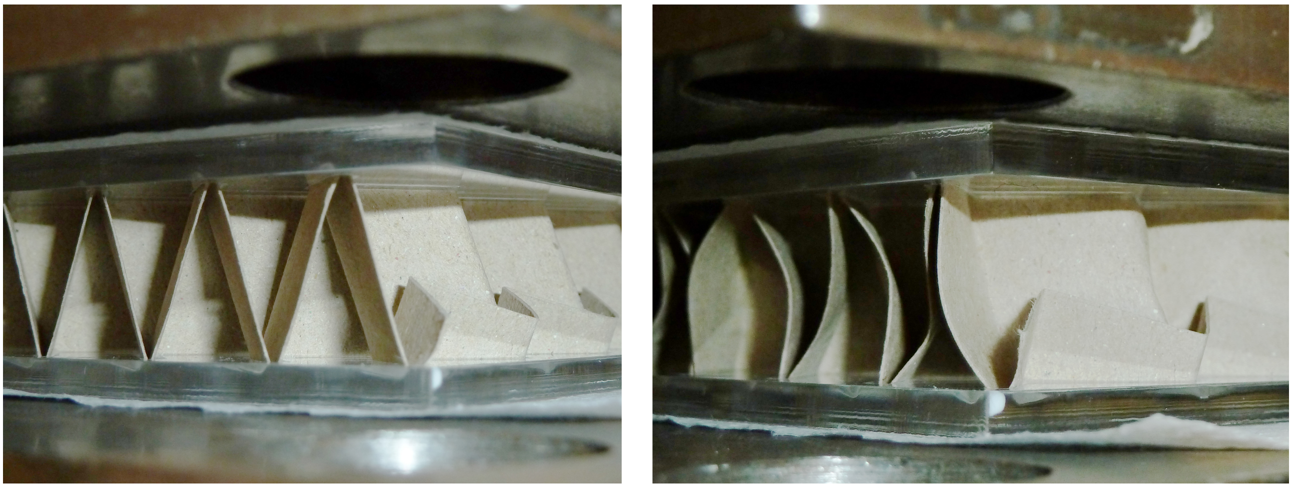


Figure 1: Compression test of a miura foldcore sample with typical buckling failure mode.

Wearable Metal Origami

Dr Tine De Ruysser for the 6th OSME Conference

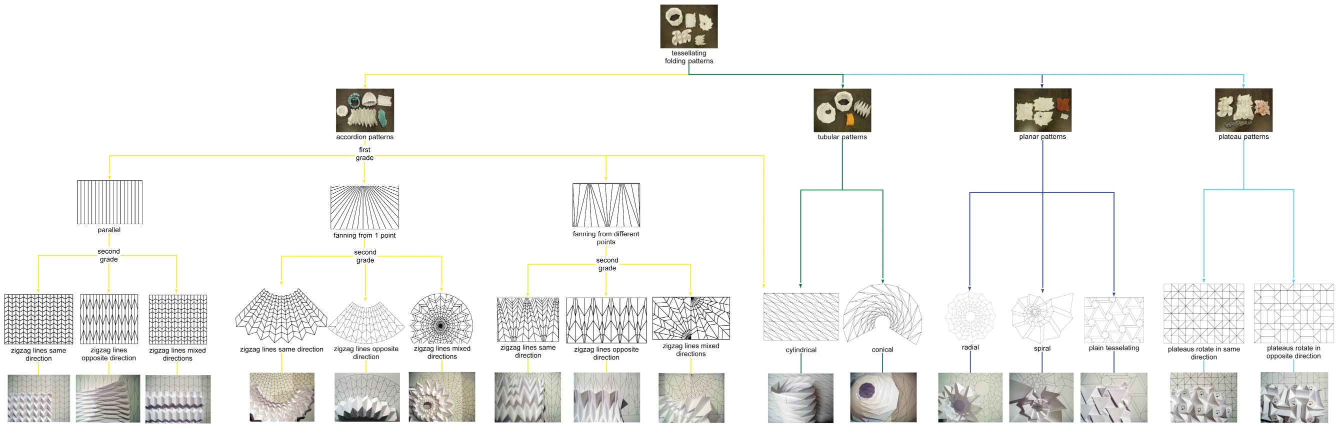
The art of paper folding has possible applications in design and engineering, but paper itself is often too weak as a material for real-life applications. Durable materials that can be folded with the intricacy of origami patterns are needed, but not easily available. This paper describes one way of making a textile-metal laminate, developed especially for the creation of tessellating origami structures. The material draws upon the knowledge of various design fields and is suitable for the creation of different types of objects. It is particularly relevant to jewellery for its visual character; its flexible, almost organic movement that easily adjusts to the human body; and the possibility to use precious metals.

The material is made by applying metal platelets to a sheet of fabric to mimic the unfolded parts of the paper. In the location of the folding lines the fabric is left bare so it can function as a hinge. The process used for the application of the platelets is the electro-chemical process 'electroforming'.

The combination of metal with textile hinges means that apart from a basic understanding of the mathematical principles of origami, other elements have to be taken into consideration. Thickness of the material becomes important (1), as well as the combination of weight, flexibility and gravity. Designs have to be user-friendly for wearers that are not origami experts, so patterns are chosen to work with the movement of the body, edges have to be finished, and constraints are introduced to ensure the designs cannot completely unfold.

The material could only be developed because of a multidisciplinary approach, revolving around tessellating origami and incorporating printing, electroforming, design methodology, CAD drawing as well as the study of material properties and existing folding and flexible structures.

The exploration of origami patterns was the first pillar of the research. I made my own classification of folding patterns, based on how their movement when (un)folding to help with the design of wearable pieces. I chose to use a family tree as classification system, which will be refined as my knowledge of tessellating origami improves through newly available software(2)(3) and attendance of the conference.



The three other pillars for the project were technical development of the production method, the exploration of design strategies and the placement of the project in the context of natural and man-made structures and objects. All these will briefly be touched upon, with a focus on how skills from different disciplines were brought together.

When presenting my paper at 6OSME, I will show samples of my work and let the attendees handle the pieces.

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Point of View: Joys and Uses of 3D Anamorphic Origami

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Finished origami in the artistic and recreational sense is a very visual thing. Unless touching is allowed, the shape together with material and lighting generates the primary impression of an origami piece. With any amount of colored, bi- or multicolored and patterned paper available this allows for very appealing and surprising end-results. Mostly though, the resulting pattern and visual impression is defined by the folding pattern.

In reversal of this process, it is also be possible to define a desired visual impression that is projected by the finished piece and to reverse-engineer the developed image that needs to be applied to the flat pattern. In general, this process is termed anamorphosis and describes a (distorted) projection that has to be viewed through a certain device or from a certain position to reconstitute the desired image.

By using origami, or more specifically three-dimensional tessellations as such an anamorphic viewing device, it is possible to generate interesting and fascinating interactive visual impressions. Because tessellations often exhibit a large number of spatially precisely oriented faces, the computation of the unfolded anamorphic image is a very manageable and rather straightforward process (Figure 1).

This paper will explain the principle of anamorphic origami (tentatively termed *anam-ori*) using periodical tessellations and will demonstrate several interesting and possibly entertaining effects and results that can be achieved using this method. Last but no least potential uses of this method will be discussed.

Keywords: Origami, Anamorphosis, Tessellations, Design

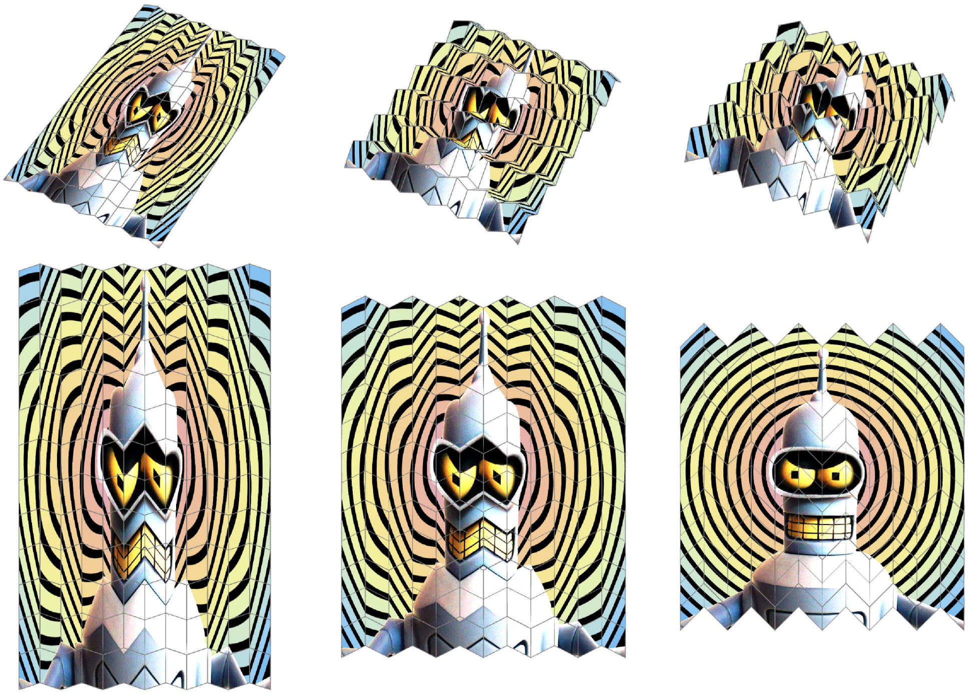

Figure 1: Anamorphic origami. The distorted flat image is transformed by the folding process to show the intended image. The tessellation shows the intended image only from one specific point of view.

ABELIAN AND NON-ABELIAN NUMBERS VIA 3D-ORIGAMI

JOSÉ IGNACIO ROYO PRIETO AND EULÀLIA TRAMUNS

It is well known that the description of the origami folding operations via the Huzita-Justin axioms leads to the set of points constructible with origami, which is the smallest subfield $\mathcal{O}$ of the complex plane which is closed under square roots, cubic roots and complex conjugation (see [A00]). Those axioms form a complete set of axioms in some sense (see [AL09]), but if we assume more moves axiomatically, the scope of the constructible points can be extended. For example, if alignments involving arbitrarily many simultaneous folds at the same time are permitted, it has been shown in [AL09] that Lill's method allows one to construct any real algebraic number.

In this work we intend to push forward the arithmetic limits of $\mathcal{O}$ by introducing some manoeuvres which involve the use of the third dimension, that is, allowing the paper not to remain flat after being folded, but in such a way that actually determines new points. The moves we propose are easy to perform physically, and consist in a simple usage of a flat surface, say the table, and the rigidity properties of polyhedra. With those moves, we show that it is easy to obtain all the Abelian numbers, that is, the algebraic numbers whose Galois group is Abelian, and some other non-Abelian numbers, too. We discuss the formalization and axiomatization of those 3D manoeuvres and explore the limits of what can be achieved with those constructions, posing some open questions for further research.

Key words: Origami construction, Abelian number fields, Rigidity of Polyhedra.

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Using the Fibonacci Series to Fold a Golden Rectangle

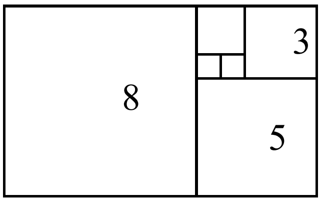
Annette Hatch
University of New Mexico – Valencia Campus

In recent years, there has been an effort to move the learning of mathematics away from rote problem solving to a more investigative approach. In an effort to increase the interest of my students in the beauty of numbers, I have led many explorations of the patterns found in simple numbers. Even students not enamored with mathematics are thoroughly fascinated by the relationships that they discover.

One activity of this nature I have used involves a connection between the golden ratio and the Fibonacci series. By looking at the proportions between consecutive Fibonacci numbers, one discovers a convergence toward an approximation of the golden ratio, φ:

$$\varphi = \frac{1+\sqrt{5}}{2}$$

It is usual to create golden rectangles and the familiar spiraling "nautilus" layout using a compass and a ruler. But it is also possible to observe the layout of successive approximate golden rectangles using the well-known layout of Fibonacci number-sized square tiles as shown in the figure below.



This presentation will demonstrate the use of origami to fold successive approximations of golden rectangles similar in design to the figure above through the use of Fibonacci numbers and simple origami square grids. The audience will be provided materials and will be led to the completion of at least one of the simpler variations.

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Equal Division of Any Polygon Side by Folding

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In origami, Haga's theorem, crossing-diagonal, Fujimoto's method, and many other methods exist for dividing a square into N-th equal parts without using a ruler. All these techniques are useful when the folder begins with a square. But what if the shape is not a square? After the first few folds, most models are no longer squares. The needed division may be on one of the non-square flaps. In order to divide non-square shapes, the author has devised a systematic approach, the median binary method, to fold any polygon side into equal divisions.

This generalized method can be proved using similar triangles. The procedure starts with a few binary folds on a chosen median line adjacent to the target side of the triangle. An extension line connects the apex opposite the target side to the newly created division along the median line. This extension line divides the target side into one of the desired N-th divisions (Figure 1). This method can be easily applied to any polygon, since a polygon is composed of multiple triangles (Figure 2).

With practice, the median binary method can be easily memorized and used without diagrams or other references at the beginning or middle of the folding process.

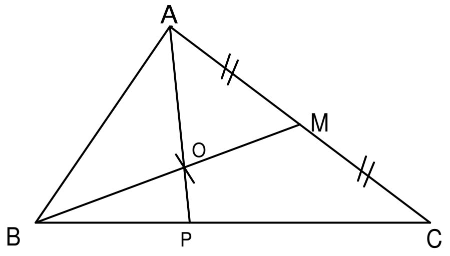

Figure 1. Locating one of the N-th division points, P. target side, BC; midpoint, M; one of the binary division points, O.

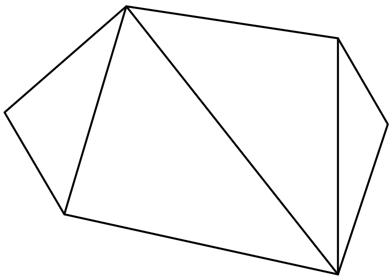

Figure 2. A polygon is composed of many triangles.

Planning Motions for Shape-Memory Alloy Sheets

Daniel Tomkins, Mukulika Ghosh, Jory Denny, and Nancy M. Amato

Abstract

Shape Memory Alloys (SMAs) are smart materials that remember their original shape, such that a deformed SMA will return to its original state upon application of heat. Due to this reconfigurable property, SMAs are used in aeronautics, medicine, and other fields where dynamic re-engineering or actuation of components is required. In this work, we consider an SMA material inlaid in a compliant passive layer to form a sheet that folds on application of heat [1, 3].

Planning the motions of these SMA sheets poses an interesting computational challenge. In general, motion planning is the problem of planning an object's (e.g., a robot) motion from a given start position to a goal state. For an SMA robot, the motion is planned from an unfolded flat state (Fig. 1(a)) to a folded shape state (Fig. 1(b)). We consider robotic systems composed of SMA sheets (used as joint actuation mechanisms) connecting rigid components.

Sampling-based motion planners [2] are considered to be the state-of-the-art due to their superior ability to solve high dimensional problems. This work describes how this SMA folding problem is reduced to a sampling-based motion planning problem. One of the major challenges in using a sampling-based planner for an SMA sheet is that the sheet can bend in an infinite amount of directions. However, because these sheets are only used as joints, we restrict the motion of each sheet to a uniform radius of curvature along the length of material which constitutes the joint; this is represented by a single degree of freedom, or parameter. In addition to modeling the system, we consider the problem of finding not only collision free but also gravitationally stable motions (Fig. 1(c)). This additional feasiblity makes planning particularly difficult.

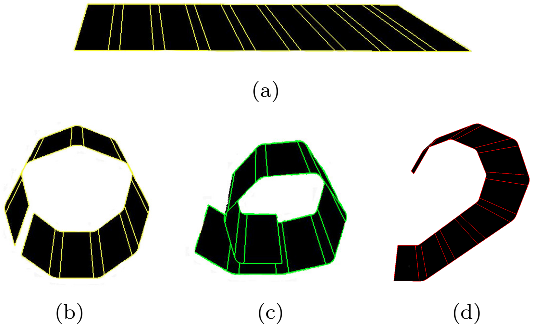

Figure 1: Folding of SMA robot from unfolded flat state (a) to folded shape (b) consists of feasible intermediate states (c) which are non-colliding and gravitationally stable unlike (d).

With these restrictions and constraints, we show how we can apply a specific sampling-based motion planner, Rapidly-exploring Random Trees (RRT) [2] to this problem. We adapt the original algorithm with a new distance computation for our SMA robots and a rigidity analysis to see which components can be moved without causing instability (as in Fig. 1(d)).

Our results validate our model of these SMA sheets and our new motion planning algorithm for such robots by folding interesting 3D shapes composed of SMA sheets and rigid components. Our results show significant flexibility in modeling various planning problems, and significantly improved motions compared to not using stability constraints.

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The effect of imagery controllability on Origami performance

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The present study examined the effects of imagery ability on Origami performance. In our previous pilot case study, participants made a "crane" utilizing an Origami diagram. Furthermore, participants' imagery controllability and imagery vividness were measured by the Mental Rotation Test (MRT; Vandenberg & Kuse, 1978) and the Vividness of Visual Imagery Questionnaire (VVIQ; Marks, 1973) respectively. The results showed that the high MRT scorer took a shorter time to make a crane than the low scorer, but imagery vividness had no effect on their performance. It could be interpreted that only imagery controllability relates to understanding how to make an Origami figure. It was possible, however, that the diagram helped participants to generate detailed images of the intermediate-stages of the figure, and that they only had to mentally manipulate an imagined paper to understand the figure's transformation from one state to the next. Consequently, imagery vividness might not have been reflected in performance. If such is the case, when direction for making a figure is displayed without diagrams, participants have to generate the detailed image of the paper, and imagery vividness is reflected in performance.

Thus, in the present study, we adopted the video showing no paper, but only hand movements creating a "samurai helmet". Eighteen female and 13 male undergraduates participated in the experiment. According to their self-reports, they had not had origami experience for, at least, the last 4 years. First, they took the VVIQ and MRT. Four or seven weeks later, they made a "samurai helmet" by following the video mentioned above. The required times to complete the figure (Time) were recorded. After that, they reported how many kinds of figures s/he had ever made. This reported value (EXP: experiential number) was considered as their background in folding.

We conducted the multiple linear regression analysis with backward elimination method covering Gender, EXP, MRT and VVIQ scores as explanatory variables, and Time as the objective variable. In the results, we found the significant model containing MRT score, EXP, and Gender (adjusted $R^2 = .473$, $p < .001$). This suggests that females completed the figure faster than males, and that the past experience of Origami facilitated the performance. Additionally, the higher MRT score a participant scored, the shorter time s/he took to complete the figure. On the other hand, no effect of imagery vividness was indicated. These results suggested that at least imagery controllability is important for understanding how to make an Origami figure, and the effect of imagery vividness should be continuously investigated in future research.

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Toward optimization of stiffness and flexibility of rigid, flat-foldable origami structures

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Abstract:

Origami patterns have been applied in spatial structures to make stiff shell structures as well as flexible transformable systems. Folding a planar sheet into a 3 dimensional configuration changes the stiffness and the behavior of the sheet. The design of folding patterns can change such properties. At times, origami structures need to simultaneously possess stiffness and flexibility. For example deployable structures in architecture and engineering need to follow a flexible prescribed deployment mechanism, but they need to be stiff for externally applied loads.

The objective of this paper is to enable the design of rigid-foldable, flat-foldable origami patterns that have desired stiffness properties, by using structural analysis and optimization. In order to achieve this goal, we first propose a simplified scalable analytical model that simulates the behavior of the thin material. Our model extends an existing formulation (Schenk and Guest 2010) to be geometrically scalable, and incorporates the thickness to length ratio of the material. We then investigate the stiffness and flexibility of one degree of freedom (DOF) rigid-foldable origami tubes (Tachi and Miura 2012). The stiffness and flexibility of the tubes can be represented by the eigenvalues and corresponding eigen-modes of the structure. The first six eigen-modes correspond to rigid body motion in 3 dimensional space, the next mode corresponds to the rigid folding motion, and subsequent modes represent structural bending and deformation. We parameterize the tube by the sector angle, length proportion, and thickness, so that the tube remains rigid-foldable, and we use a folding angle to define the configuration. By changing the parameters, we can minimize or maximize an eigenvalue, making the structure respectively more flexible or stiff, for the corresponding mode. In this paper, we maximize the eigenvalue band-gap between the seventh and the subsequent modes. This makes the structure easier to fold and deploy in the rigid folding mode, while at the same time stiffer against bending modes for external loadings.

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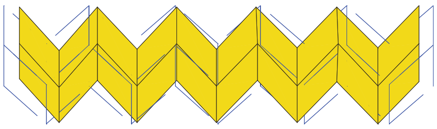
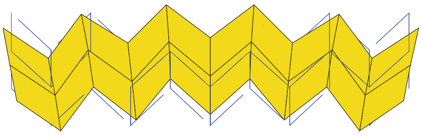


Mode #7: Rigid folding motion of tube (dotted outline shows original position of structure)



Mode #12: Bending mode of tube (dotted outline shows original position of structure)

Seven Year Study of Origami’s Impact on Spatial Skills of College-Age Students

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Spatial visualization is the ability to mentally manipulate 2-dimensional and 3-dimensional objects. It is an element of human intelligence (Gardner, 1993; Piaget & Inhelder, 1956) and an important skill within the study and learning of mathematics (NCTM, 2000; NGACBP, 2010). Research on methods targeting the strengthening of spatial skills has shown it can lead to a deeper understanding of mathematical concepts from elementary through college age individuals (Newcombe, 2010). This paper presents a seven year, longitudinal study on the impact of one such method targeting the improvement of spatial visualization, namely Origami.

Research presented is gathered via a four credit college-level course dedicated to the practice of Origami called the *Art and Math of Origami*. In this course, college level students study the art of paperfolding in depth including an infusion of mathematical concepts within instruction. As part of the course students are pre- and post-tested on their spatial visualization skills utilizing excerpts of a field-tested national intelligence test (Ekstrom, 1976). Data is collected on three tests related to spatial ability (see Table 1). Initial analysis of data collected over three years’ time has proven that “regular and repeated exposure to origami benefited both the spatially strong and weak college age students” (Author, 2011). This paper further extends findings over the seven years this course has been offered to date. The sample size includes 164 students having taken this course from a variety of disciplines. Analysis includes consideration for pre-existing spatial skills students may have. While this study has no comparison population by nature of its design, analysis includes considerations for pre-existing spatial ability and majors that may predispose students to this mathematical skill (ie. math and science majors). Data analysis is promising with students showing significant improvement in all three areas tested (see Table 1). Results controlling for existing skills and/or college major show some influence on gains earned on tests. However, all students regardless of background remained the same or improved abilities. Thus, overall findings of research presented provide substantial evidence that Origami is an effective tool for improving students’ spatial skills (Author, 2006; 2009; 2011).

Table 1. Summary Spatial Test Results

Test *n*=164	Pre-test mean (SD)	Post-test mean (SD)	Gain
Card Rotation	110.73 (26.88)	126.85 (25.44)	16.12*
Paper Folding	11.92 (3.34)	13.58 (3.31)	1.66*
Surface Development	38.71 (14.19)	46.01 (12.61)	7.30*
* $p<.001$			

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A study on crash energy absorption ability of lightweight structure by using truss core panel

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1. Introduction

Because of the earth environment and energy problem, the lightweight structures (e.g., honeycomb structure) are becoming a big research challenge. The truss core panels as light saving structure which can be developed by origami forming have better aspects in bending stiffness and in shear strength[1].

2. Truss core panel structure and honeycomb panel structure

In order to investigate the crash characteristics of the proposed truss core panel structure comparing to the honeycomb panel structure, absorption energy of these two structures which calculated from crash analysis(as shown in Fig.1) are compared and the optimal design method[2] is adopted to improve energy absorption ability considering probable design parameters. The truss core panel structure and the honeycomb panel structure are shown in Fig.2 and Fig.3 respectively.

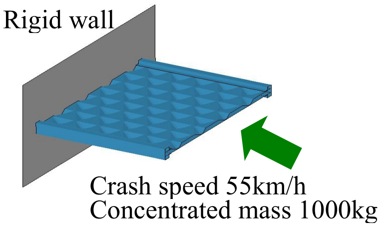

Fig.1 Crash analysis model.

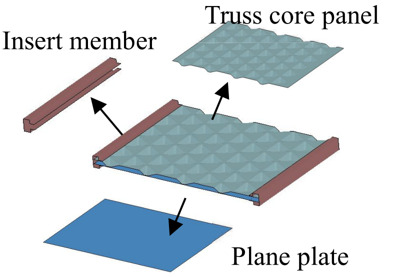

Fig.2 Truss core panel structure.

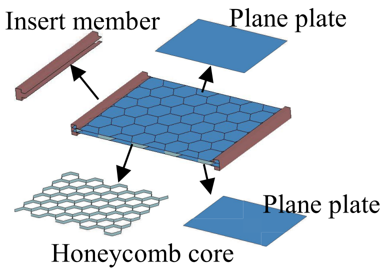

Fig.3 Honeycomb structure.

3. Comparison results

Table 1 shows that the optimal truss core lightweight structure is capable of absorbing crash energy more than that of the optimal honeycomb panel structure at the same mass.

Table 1 Comparison of absorbing energy character between truss optimal panel and honeycomb optimal panel structure.

	Absorbing energy （MNmm）	Mass （kg）	Absorbing energy per mass （MNmm/kg）
Truss optimal panel structure	17.37	3.2	5.43
Honeycomb optimal panel structure	14.65	3.2	4.58

4. Conclusions

(1) In order to improve the absorb crash energy as much as possible, the truss core panel structure and honeycomb panel structure are introduced into one proposed structure which may be used as the flat floor in the vehicles. And the response surface method is used to optimize the two structures.

(2) According to the optimization results, the truss optimal panel structure is capable of absorbing energy more than the honeycomb optimal panel structure at the same mass.

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Creating Novel Motions with N-Long Chains of Interconnected Spherical Mechanisms

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Rigid Kinematic Origami (RKO) can be modeled as series of interconnected spherical mechanisms [1]. Recent work in classifying [2] and predicting [3] the motion of origami mechanisms has laid the foundation for creating new action origami models with complex and novel motions.

Most RKO models rely on simple coupled mechanisms or large tessellated arrays of mechanisms. The design space of tree-shaped graphs of origami mechanisms is mostly unexplored [2]. Each tree is composed of smaller n-long chains of mechanisms; by studying the motion of these chains it is possible to model the motion of the whole tree.

We will show an approach for creating novel motions and a method for generating crease patterns of chains of mechanisms with a specified output by studying the two-dimensional motion of chains of mechanisms. We will also explore potential applications of these novel motions. By applying this method to origami models designed from trees it is possible to create lifelike motions in action origami.

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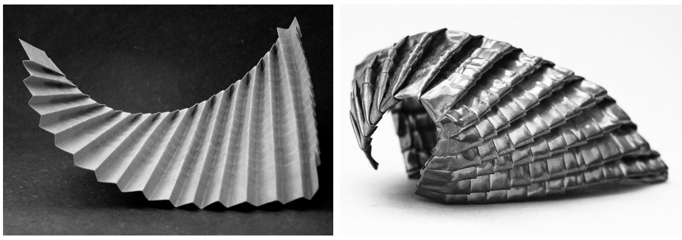
On pleat rearrangements in pureland tessellations

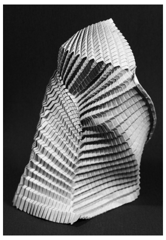
Goran Konjevod

May 30, 2014

Since 2005 I have folded hundreds of pureland tessellation models based on the alternation of simple horizontal and vertical pleats originally introduced by Paul Jackson. Most of these foldings are constructed on a uniform square grid and the pleats all follow gridlines. Despite this basic simplicity, variations in the pleating order lead to different tension forces in the folded sheet, which in turn cause curvature in the folded sheet. Thus, a simple repetitive process of folding straight lines leads naturally to a broad range of three-dimensional folded shapes.

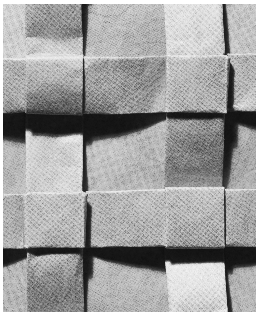

Basic form (Paul Jackson) in its unlocked and locked forms;

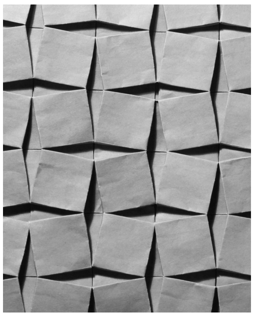

Double wave (2007);

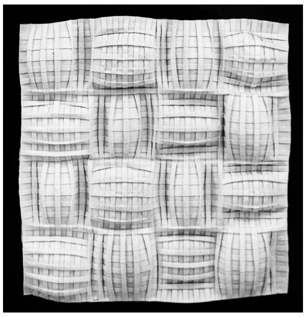

Cliff (2009)

In 2008, Andrew Hudson posted on Flickr examples of a more general approach that, in addition to simple pleating, includes the step of rearranging some pleats after the folding. No new creases are created in this process, but some change their type. This technique allows an easy way to produce some interesting visual effects. The simplest is a cyclic sequence of pleats p_1, p_2, p_3, p_4 in which p_1 apparently must be folded before p_2, p_2 before p_3, p_3 before p_4 and p_4 before p_1. (Clearly, one way to do this is to fold them in order $p_1, \ldots, p_4$ and then rearrange the intersection of p_1 and p_4 so that p_4 appears to have been folded before p_1.)

For example, fields of apparently twisting squares can be folded without making a single twist fold.


1-rearrangement pleat cycle;


No-twist field (reverse);


Multiweave pattern (32×32);


Change study No. 6 (64×64)

Over the past few years I have used this pleat rearrangement approach on several occasions and immediately realized that there is a natural optimization problem associated with it. The number (and nature) of pleats is fixed by the goal design, but the order in which the pleats are folded may change the number of rearrangements that must be performed. Since folding the rearrangements can be tricky, especially in the middle of the sheet, minimizing the number of required rearrangements may reduce the amount of work in folding the model. This observation motivates a discrete optimization problem: find the order of pleating that minimizes the number of necessary pleat rearrangements. This problem turns out to be equivalent to an optimization problem that has been studied in the computer science literature, namely the *feedback arc set problem in bipartite tournaments*. The results in the literature show this problem to be NP-complete and also give a 4-approximation algorithm, that is, a heuristic with the guarantee that the solution it produces requires no more than 4 times the optimal number of rearrangements. In contrast, I will argue that a large class of designs exhibiting repetitive behavior (and thus of interest in folding tessellations) is easier to solve.

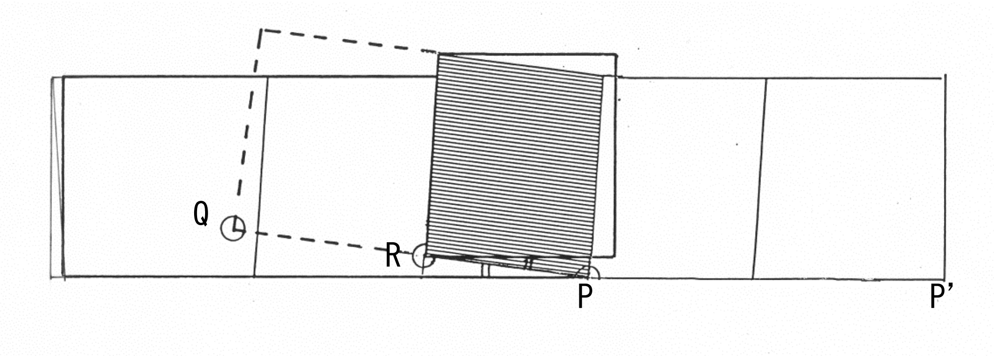
Three-point method of folding rectangular paper into Miura-ori
– Design of mini paper media for education –

Koryo Miura* and Naoko Tsuji**
University of Tokyo*, Handa Sky Science Museum**

This work is to present a method of manual folding a rectangular sheet of paper to Miura-ori paper media, without using a fold-line chart. The principal procedure is based on the three coordinates points preliminarily marked on a sheet of paper.

SCHEME: Paper size A4, 5 lateral divisions and 3 longitudinal divisions.

Fig. 1



1. Refer to the mark P, fold the paper to the lateral strip consist of 3 layers.
2. Fold the strip so that the right lower corner P’ exactly meets the mark Q.
3. Fold back the strip at the mark R so that the lower edge is parallel to the initial edge.
4. Notice that the basic parallelogram is formed (with shade) inside the folded strip. Turn it over. Fold the right side of the strip along the longitudinal sides (at mark R and then P) of the basic parallelograms. As the result, the complete fold lines of Miura-ori are created.
5. For Maura-ori, zigzag mountain folds lines and valley fold lines should appear alternately. Adjust the folds following this rule to approach the final result.

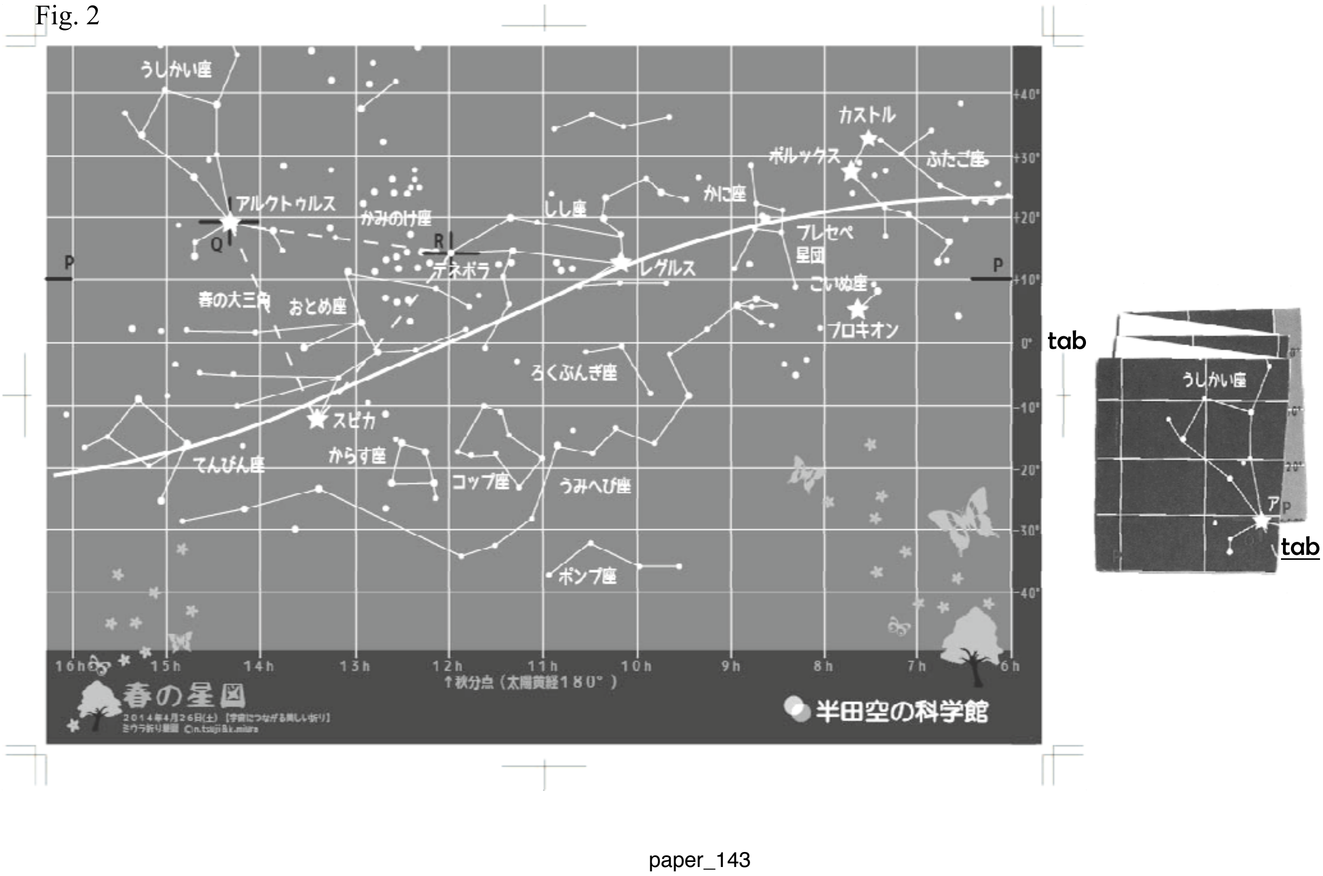
*1.The coordinate data P, Q, R, refer to the lower left corner of the sheet is:

P, P’ (---, 140), Q(53, 157), R(118, 148) (mm)

*2. Number of both lateral and longitudinal divisions should be odd numbers for positioning of pull tabs.

EXAMPLE: Sky Atlas by Handa Sky Science Museum, Japan.

Fig. 2



6OSME Abstract

Submission from Chris K. Palmer
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Paper and Textile Folding

A Synergistic Relationship

An examination of the differences, similarities and relationships between paper folding and textile folding. The example of an Eight-Fold Flower Tower will be analyzed showing the paper folding method, the method of deriving a pattern for execution in textile using the paper model and the technique for folding in textile. Other patterns will be examined similarly flowing from paper to examined paper model to pattern for textile folding.

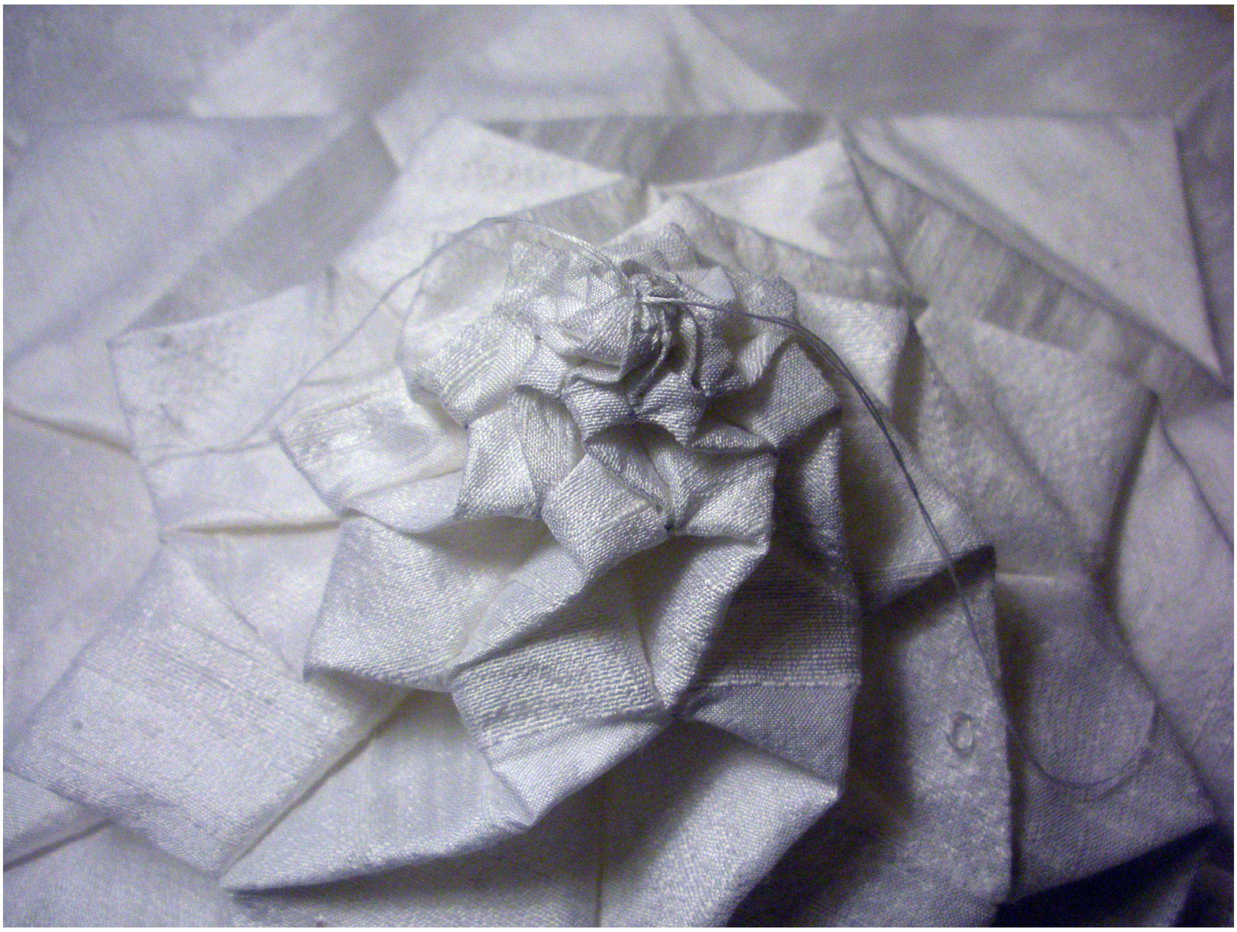

Figure 1
Eight-Fold Flower Tower - Silk

The Elusive Technique of Folding Anatomical Subjects

Christopher Yuki Itoh

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Biology, Johns Hopkins Bloomberg School of
Public Health, Baltimore, MD, USA

Over the centuries, human anatomy has proven to be a compelling subject for visual artists. Representing anatomy accurately is a goal that has been sought by artists and anatomists such as Leonardo da Vinci (who made important discoveries in anatomy), Andreas Vesalius (the founder of modern human anatomy), and Frank Netter (a surgeon and a medical illustrator). However, depicting anatomy accurately using origami has not been fully investigated. Anatomical subjects pose a unique challenge to paperfolders because of their asymmetry, color changes, surface area limitations, and the inclusion of various details such as foramina and depressions. Drawing upon my training in human biology, I have explored how to accurately represent these elements, specifically in the heart, brain, vertebral column, and skull (figure 1). I have observed not only that origami represent anatomical models very accurately in 3-dimensions, but also that the models can be shown through time—essentially, 4-dimensional models. My current origami research involves a project to incorporate LED lights and contractions systems to represent the electrophysiology of the heart (figure 2) as well as investigate the parallel between the process of origami folding and embryology. Because of the low cost of paper, the use of origami to model anatomical subjects may benefit the teaching of anatomy and physiology where school funding does not provide for plastic models or cadavers. In addition, because new advances in medicine challenge medical illustrators and model creators to innovatively design, the hands on and more artful nature of anatomy origami may aid in health education and communicating ideas to patients and the general public.

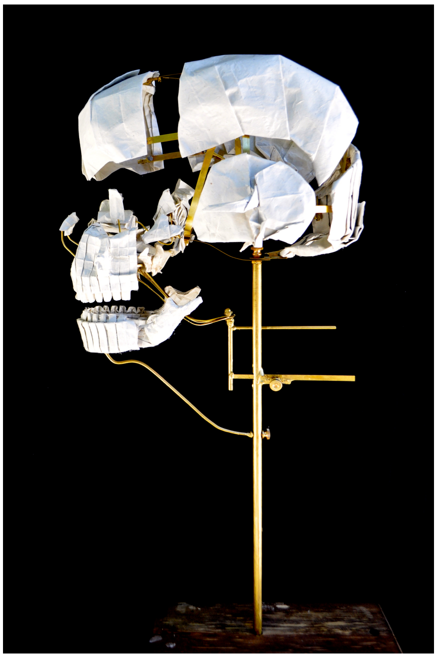

Figure 1: Beauchene Skull, a disarticulated human skull that has been reassembled on a stand, allowing for complex structures to be studied individually

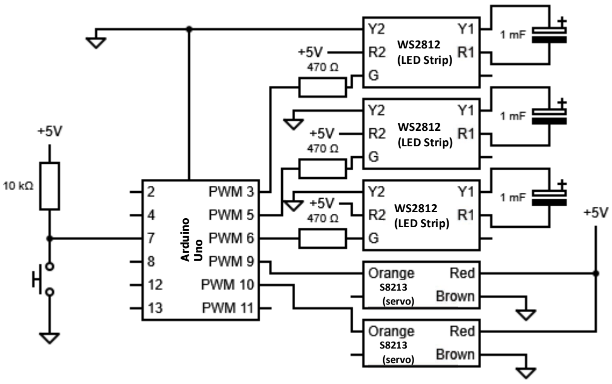

Figure 2: Circuit diagram for servos and LED lights on the contracting origami heart model

Self-Deploying Origami Models Using Misaligned Crease Patterns

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This work illustrates a new strategy for designing self-deploying origami actuated by the elastic energy stored in its own facets or simple formed actuators such as springs. Traditionally, origami-based structures are designed on the premise of "rigid folding," i.e., the facets and fold lines of origami can be replaced with rigid panels and ideal hinges, respectively. However, from a structural mechanics viewpoint, these systems are usually overconstrained and have negative degrees of freedom (DOF), i.e., the number of constraints exceeds the number of variables. In these cases, the singularity in crease patterns guarantees their rigid foldability. Figure 1 shows the 1-DOF origami model with two hollow facets A and B. If the crease pattern has the aforementioned peculiar rigid foldability, the hollow facets will not deform during folding/unfolding of the whole structure (right). Next small misalignments are introduced in the crease lines. Obviously, these misalignments break crease-pattern-dependent rigid foldability. However, because of its 1-DOF mechanism, achieved by the existence of hollow facets, the structure can be folded/unfolded the same way as with the correct (without misalignment) crease model. The important difference is that hollow facets with a misaligned crease pattern will deform during folding/unfolding of the whole structure (left), unlike with the correct crease model. In the proposed method, deployment of the structures is controlled by the deformation of the hollow facets. First, a new extended rigid-folding simulation technique is proposed to estimate the deformation of the holes. Next by using the above technique, the proposed method is applied on arbitrary-size quadrilateral mesh origami. Finally, by using the finite-element method, the authors conduct numerical simulations and confirm the deployment capabilities of the models.

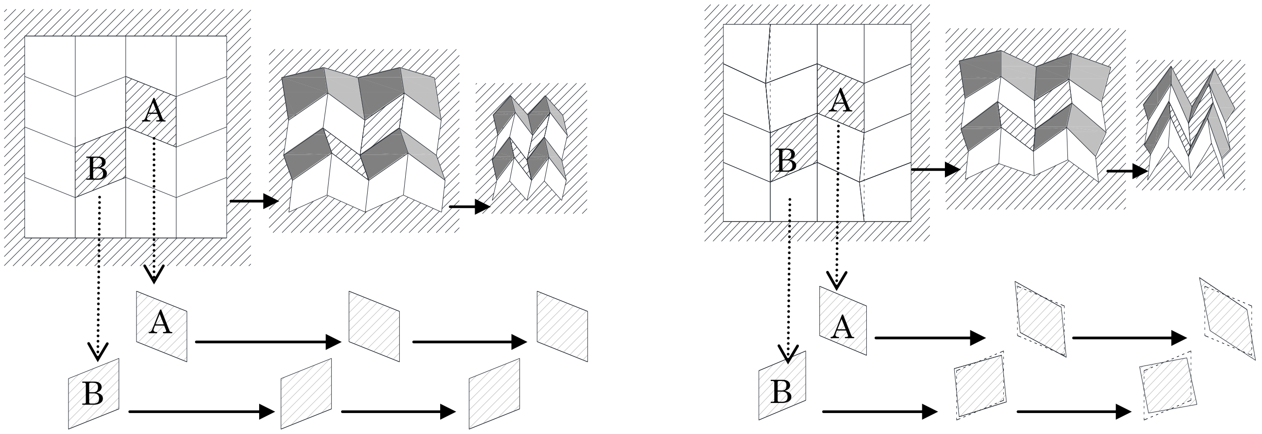


Fig. 1 Deformations of hollow facets. (Right) In correct crease patterns. (Left) In misaligned crease patterns.

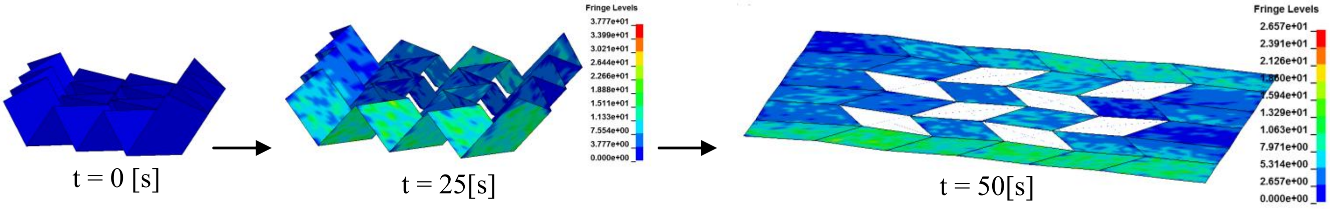


Fig.2 Deployment simulation using LS-DYNA. The advantage of the proposed method is that it requires only a simple actuator that can shrink or expand in length between two vertices. In this case, elastic spring elements are inserted on diagonal lines to deform the hollow facets.

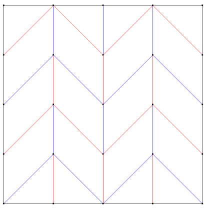
Characterization of origami corrugation crease patterns

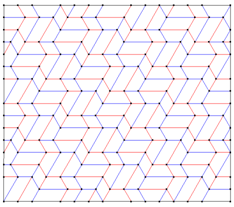
(Work in progress)

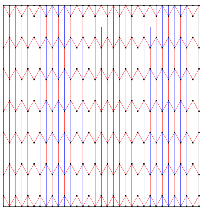
Ray Schamp and Goran Konjevod

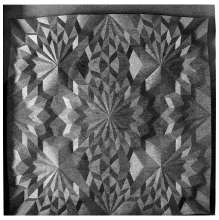
Origami corrugations are a well-known class of origami model but have never been formally defined. We present work towards proposing a definition that captures a large subset of origami corrugations.

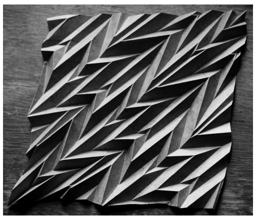
The intuitive notion of an origami corrugation is a repeating pattern that alternates mountain and valley folds. Many corrugations are relatively flat, due to the regular alternation, but they tend to be flexible and can often be deformed into curved surfaces. Some, but not all origami corrugations are flat-foldable. An additional property of most corrugations is that they may be unfolded relatively easily. In fact, many corrugations are rigidly foldable.


Miura-ori


Equidistant weave


Curl


A radial corrugation


A degree 3 corrugation

We have been investigating several local conditions that embody properties seen in typical corrugation patterns. These conditions include:

- Vertex alternation: every "out" vertex (with $M_v > V_v$, where M_v and V_v are the number of mountain and valley creases, respectively, meeting at v) has an "in" vertex ($V > M$) neighbor. This condition appears to be natural, but is not valid for some patterns we consider corrugations, such as the Curl pattern shown in the Figure.
- Vertex edge count: $|M - V| \in \{0, 2\}$ at every internal vertex. This is a modification of Maekawa's theorem that allows an equal number of mountains and valleys, in order to include central vertices in radial corrugation patterns (see Figure). This condition allows in particular all flat-foldable origami patterns.
- Vertex angle condition: consider two creases c_1 and c_2 that share both an endpoint and belong to the same face (that is, consecutive creases meeting at a vertex). If both c_1 and c_2 are of the same type (both mountains or both valleys), then the angle between them is at least 90°. If they are of opposite type, then the angle between them is at most 90°. This condition excludes faces wrapped around other faces.
- No extreme faces: the boundary of every internal face contains both a mountain and a valley crease. Only a few types of patterns that we consider corrugations seem to violate this condition.
- Face alternation: every face neighboring an "out" face (one with an all-mountain boundary) has at least one valley edge. We are beginning to study this condition.

We have implemented these conditions in the form of a corrugation checker class in ORIPA and are currently working on adding to ORIPA tools for automated and semiautomated crease assignment for origami corrugation patterns. We are also working on the implementation of an algorithm for finding crease assignments that fold into origami corrugations by walking across the vertices of an unassigned pattern and applying the corrugation conditions to the faces and vertices encountered.

AN OFFSET CONSTRUCTION TECHNIQUE
FOR THICK RIGID FOLDABLE ORIGAMI

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ABSTRACT

A novel method for thick, rigidly foldable, origami-inspired design in introduced. Origami panels are mathematically assumed to be planar with zero thickness. To accommodate thick panels while maintaining the same motion as the paper model, hinge rotational axes should lie along the folds in a common zero-thickness plane. Using this new offset panel technique, arbitrary panel thickness can be used. The construction begins with a rigid-foldable model's panels stacked in order in their closed or stowed position. A joint plane is chosen to represent a zero-thickness virtual plane and each panel is connected to the chosen plane with vertical segments. The joint plane is not limited to panel surfaces and can even lie far offset from the stacked panels. The extension segments from the stowed panels rigidly connect the panel to a joint with its rotational axis in the chosen joint plane. When using planar panels, the offset panel technique requires that the origami fold pattern be rigid foldable and sortable, however, by using tilted or nonplanar panels, sortability may be relaxed. The length of a vertical segment is determined by the distance between each stowed panel and the chosen joint plane. Multiple panel thicknesses can be accommodated within the same model. Clearance holes may be required to avoid self-intersection of joints or panels. This technique can yield compact stowed structures that preserve the origami souce model's kinematics with full range of motion. The technique is demonstrated using well-known origami patterns in various configurations.

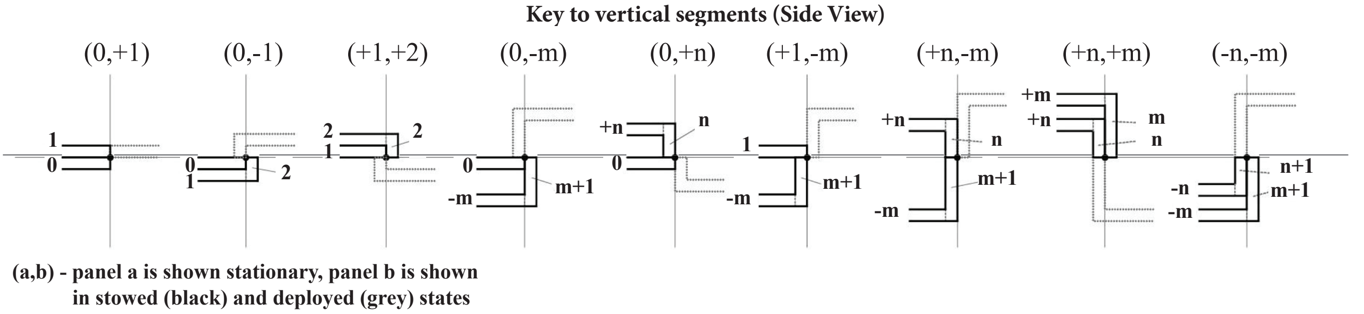

Figure 1 - Depending on the panel order and separation, the hinge placement varies. The side view shows possible placement options with panel/hinge relations. The solid lines show the stowed position while the dotted lines show the open position. The black dots indicate the rotational axis of the hinge. The left 6 orientations are special cases when the chosen joint plane is the top surface of panel 0 (bottom of panel 1). When the joint plane lies offset from both panels, the general cases apply (right 3 orientations where n and m are panel numbers).

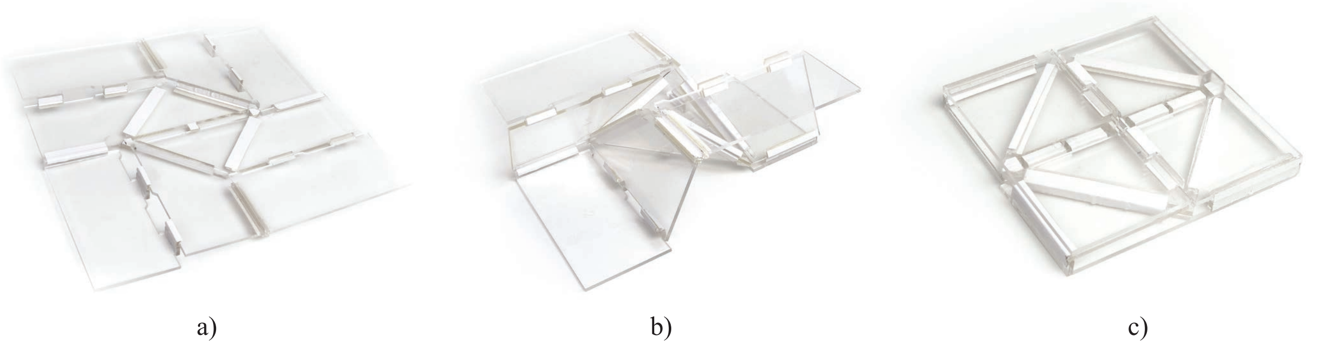
a) b) c)

Figure 2 - Square Twist (Mountain-Valley-Mountain-Valley w/ center refinement crease) constructed using the offset panel techinque. a). Fully deployed flat position with 180 degree openness (note: panels do not lie in a plane; however, all hinges are on panel 0's top surface plane). b). A mid-way stable position. c). Fully stowed initial flat position with 0 degree openness and with all hinges at the panel 0 top surface plane.

COMPARISON OF DESIGN PROCESSES IN ARCHITECTURE AND MECHANICAL ENGINEERING

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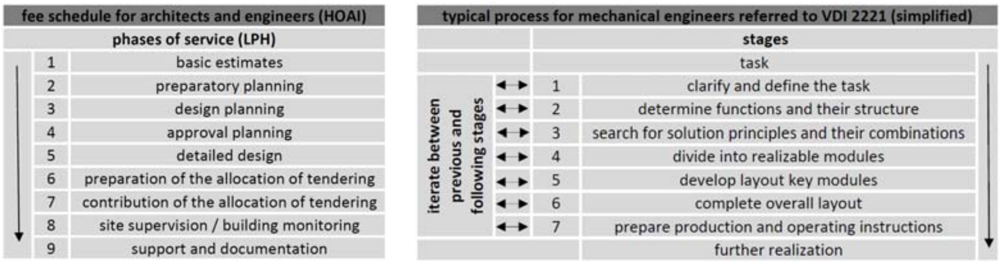
Keywords: foldable structures, domain overlapping product development, adapted design process

Developing foldable structures recommends domain overlapping teams due to the complex combination of different types of problems within those products. However different disciplines work in their specific ways based on their historical grown working processes and it is not trivial to link those developing processes. According to the design processes of architecture and mechanical engineering the distinctions between those will be exposed in this paper as these two disciplines are at least necessary for developing foldable structures for architecture applications.

Architects are focused mainly on singular solutions, which are just for one individual customer. Knowing his specific needs they need a suitable design that fits to this unique client. Usually each building project is interdisciplinary. Beside the architect dependent on the size and the complexity of the project more or less specialist engineers are involved. In the domain architecture this interdisciplinary planning process is especially in Germany regulated through the official fee schedule for architects and engineers which pretends the requested demands and an order of the planning (and realization) phases as shown in figure 1.

Products developed by mechanical engineers are usually repetition parts – in a range from small series to mass production. Even in the area of single-part production engineers try to use predefined modules that exist from earlier solutions. On this account in the field of mechanical engineering products are mainly developed customer neutral. According to the fact that most design steps are repeated methods and processes can be applied in every product development. A typical design process for mechanical engineers is shown in figure 1.



fee schedule for architects and engineers (HOAI)	
	phases of service (LPH)
1	basic estimates
2	preparatory planning
3	design planning
4	approval planning
5	detailed design
6	preparation of the allocation of tendering
7	contribution of the allocation of tendering
8	site supervision / building monitoring
9	support and documentation

typical process for mechanical engineers referred to VDI 2221 (simplified)		
		stages
		task
iterate between previous and following stages	1	clarify and define the task
	2	determine functions and their structure
	3	search for solution principles and their combinations
	4	divide into realizable modules
	5	develop layout key modules
	6	complete overall layout
	7	prepare production and operating instructions
		further realization

Figure 1: Common processes for architects (HOAI) and mechanical engineers (VDI 2221)

The results of this comparison between the design processes in architecture and mechanical engineering are a first step for a combined design process to develop foldable structures within a multidisciplinary team.

DEMANDS ON AN ADAPTED DESIGN PROCESS FOR FOLDABLE STRUCTURES

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Keywords: foldable structures for architectural and engineering applications, student projects, preliminary work for an interdisciplinary development of an adapted design process, identification and classification of typical problems

The construction principle of folding builds a predestinated alternative to established concepts when those do not provide satisfactory results for a given task. Applying folds to plates allows one to create strengthened structures that are also deployable. Due to these two key properties – strengthening and deployability – Origami-based structures are promising for architectural and other engineering applications.

Despite these obvious advantages functional realized foldable structures are rare. To make the promising Origami-based deployable structures basically usable for technical applications, especially for architecture, an adapted design process is indispensable. This adapted process does not yet exist. Also publications, in which the problems expected, the methods to be used and a reasonable order of necessary process steps are concretized are rare respectively not existent.

For the design of transformable structures for architecture applications, architects and mechanical engineers have to cooperate in a domain overlapping working group. These two disciplines typically use different design processes and it is a challenge to combine those varying developing processes. It is our aim - as a team of architects and mechanical engineers - to develop this adapted process for large-scaled deployable foldings in an interdisciplinary way to guarantee a general usability. This requires numerous preparation work. An essential task is the identification and categorization of typical problems arising during the design and construction process. As realized structures are rare, this has to be done mostly theoretically on the basis of documented design projects. Beside some published examples, student projects are suitable to identify and categorize the typical challenges. The authors performed several student projects concerning different tasks to the topic 'foldable structures for architecture and engineering' during the last years. Recurring problems have occurred during the project work which will be presented, analyzed and, if possible, generalized.

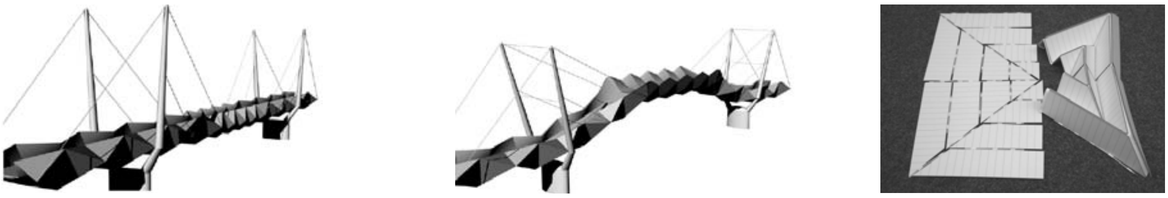

Fig. 1: Examples of student design projects

Origami beyond geometry

Goran Konjevod

May 30, 2014

Origami transforms a flat sheet of paper into a folded origami design, relying on precise folds and using paper as an ideal thin sheet. Can it be taken further? Motivated by the experiences of exhibiting folded paper pieces as works of art in galleries and museums, I ask origami designers and folders to work towards breaking the established geometric rules and constraints and I offer some of my own examples of doing so.

Most traditional origami involves making accurate straight line folds with the goal of folding a paper sheet into flat segments without stretching. Even supercomplex origami designers tend to stay within these boundaries. The most widely used exception involves curved folding, itself likely grown out of attempts at sculpting and shaping wet-folded models. I suggest that the natural next step for origami designers and folders is to go further, beyond the apparent rules inherent in the geometry of a flat sheet of paper, thus creating apparently impossible things.

Here are just a few examples of design and technique problems that seem to me natural to ask, but which I have not previously seen formulated as such. I have come across a few special-case answers to some of these questions in the origami history and literature, but they have almost invariably been sidelined as unusual or "extreme" and are rarely used. Because of the one-page length restriction for this abstract, I avoid singling just out one or two specific issues here so as not to cause any misunderstanding. In the talk, as well as the full paper, I will discuss, and more importantly, show, specific examples.

- **Fold a sheet of paper so that it can be freely sculpted into desired forms.** The motivation here is obvious: existing design techniques for origami design place a much heavier emphasis on the geometric constraints of folded paper than any other sculptural process I am aware of. One way to (narrowly) interpret this question is asking for a "universal base".

- **Do not fold slowly and accurately.** Errors accumulate in most complex origami designs in a way different from other visual arts. The influence of even minor misalignments is usually global. Does this have to be so?

- **Work at a different scale, so that paper doesn't behave like paper anymore.** An alternate formulation of this question is: Use "inappropriate" material: paper too thick or too thin; or other materials not normally considered foldable. (Another alternate formulation: Work with materials whose properties are normally considered a disadvantage: too thick or too thin; stretchy or otherwise difficult to fold "cleanly".)

The goal is, in a few words, necessarily with imprecision and slightly tongue-in-cheek, but without any intention of judging either discipline, to move origami design away from the constraints of engineering and towards the freedom of art.

Origami-based Modeling Techniques for Deployable Meandering Tubes

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Abstract

In this presentation, two modeling techniques are presented to design developed patterns for deployable meandering tubes, with the regular arrangement of fold lines based on three fundamental folding designs; the Miura folding, the bellows folding, and the folding with six fold lines at each node. The developed patterns are assumed to be made of soft materials such as vinyl and polypropylene, considering the further practical application to industry. The applicability and limitations of each folding design and the modeling techniques are discussed and summarized in this presentation. Both modeling techniques retain the regular arrangement of the fold lines so that the obtained developed patterns are as simple as possible. It can help designers' understanding and make a manufacturing process simple and efficient, because the developed patterns can be standardized if these designs are adopted into industrial designs in the future.

Technique 1: The first technique is conducted by allowing cutting and removing unnecessary parts from the original developed pattern that corresponds to a straight cylinder. First, we assume that any meandering tubes can be approximated to multiple tori connecting each other, which have the identical tube radius with the different principal curvatures, and therefore, we can start with designing the developed pattern of a torus as a unit pattern. Next, the unnecessary parts are properly adjusted to be removed, considering variable principal curvatures so that it is possible to make the develop patterns of meandering tubes with various principal curvatures (Fig. 1).

Technique 2: The next technique is to design additional fold lines on the unnecessary parts to fold them up into the tube, following the origami concepts that a spatial structure must be constructed from a sheet of paper by only folding without cutting (Fig. 2).

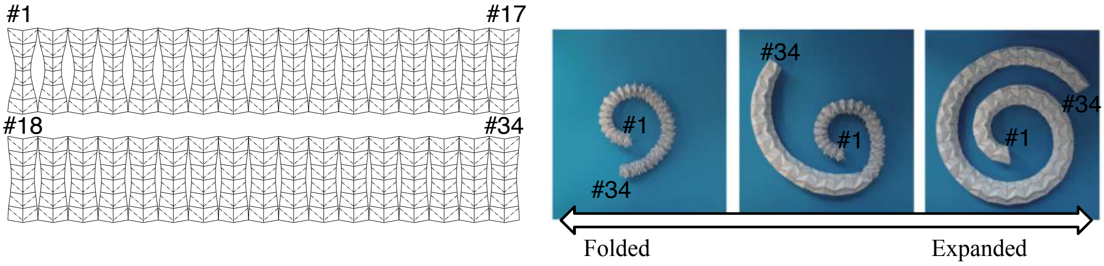

Fig. 1. Meandering tube based on an Archimedean spiral made of paper.

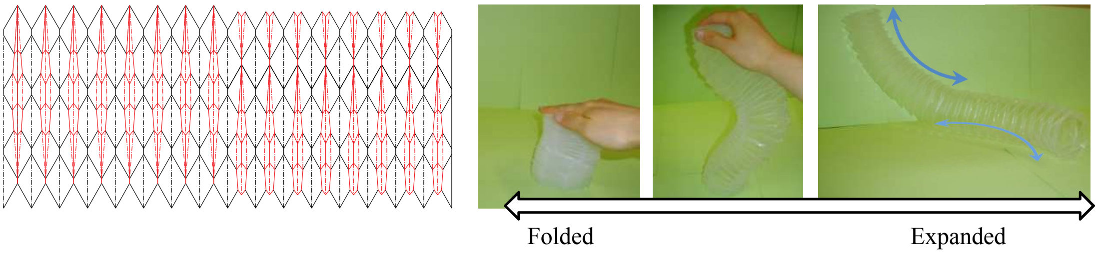

Fig. 2. Meandering tube connecting two tori, made of polypropylene sheet

[1] T. Tachi, "Freeform Rigid-Foldable Structure using Bidirectionally Flat-Foldable Planar Quadrilateral Mesh", *Advanced in Architectural Geometry* (2010), pp. 87 – 102.

[2] S. Ishida, T. Nojima, and I. Hagiwara, "Modeling Techniques of Deployable Meandering Tubes" (in Japanese), *Transactions of the Japan Society of Mechanical Engineers*, Series C, Vol. 79, No. 808 (2013), pp. 5117 – 5127.

Mathematical Models and Configuration Transformation of Origami Cartons for Packaging Industry

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Origami cartons are seen as different combinations of various shapes of panels and diverse geometric arrangements of creases with a flat foldable pattern, exploring the usefulness of origami folds for containers. As an analogy, origami folds present equivalent mechanisms in a complete innovative concept with diverse geometric shapes. Taking cardboard carton panels as rigid links, and creases as hinges, an origami carton can be regarded equivalent to a mechanism. An origami carton blank can be regarded as a combination of several spherical linkages with a number of serial chains. A cuboidal carton can be regarded as a closed-loop linkage. Cartons with complicated forms can be regarded as multi-loop linkages which result from a mixture of several serial linkages. The resultant equivalent mechanism has revolute joints only, has characteristics that the adjacent joint axes are coplanar and has such proportions that they can always be folded into flattened-configurations. Based on kinematic principles, an origami carton can be readily mapped onto a topological graph using the graph theory with crease lines being represented as edges of a topological graph and panels as nodes or vertices of the graph. Different configurations of a carton with different connectivity of panels and flaps in its various stages of packaging manipulation can be modeled with distinct topological graphs in an analytical way.

This paper presents origami models with different topological configuration states during folding manipulation and models the intrinsic relationship between discrete states analogous to the topology changes of a metamorphic mechanism having the ability of changing its structure and mobility. A matrix operation model is proposed to establish the relationship between these configuration states in origami folding and manipulation. By integrating kinematics into the topology relationship, the paper develops a hereditary digraph from the adjacency configuration matrix to generate the folding sequence which is described by a hereditary connectivity matrix. The manipulation planning is then created by a hereditary manipulation matrix that transforms the folding sequence and matrix operation model into motion of a set of robotic fingers. The mathematical models and corresponding algorithms presented a useful tool in transferring one configuration to another and gave an analytical form in developing packaging manipulation of origami cartons. The methodology is then demonstrated in a novel multi-fingered reconfigurable packaging device. This paper provides a framework for presenting the information of origami cartons and modeling origami folding manipulation.

Keywords**:** configurations, topology states, manipulation, equivalent mechanism, topological models, matrix operation, origami cartons, packaging, automation, robotic fingers.

Study on Deployable Linear Folded Stripe Structures

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This paper presents an ongoing body of research on deployable structures consisting of linear folded stripes. As, in this typology, classical, equal-sided, flat hexagons are representing a mobile mechanism, the aim of this research is to explore under what circumstances reticular structures consisting of linear folded stripes can be implemented as deployable structures.

Keywords : folding, linear folded stripes, computational methods, deployable structures

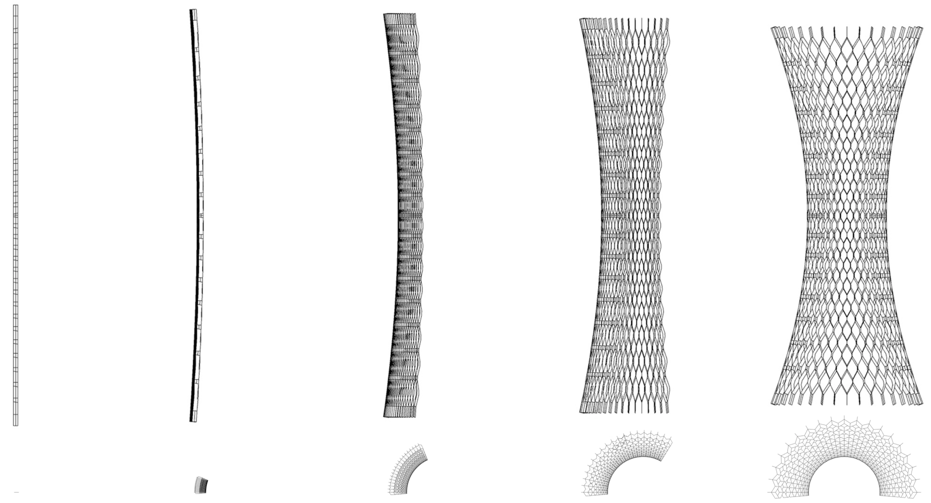

Fig 1 a deployable, rotational structure

As the stripes are rectangular in their unrolled configuration, an investigation into these structures could lead to the development of three-dimensional structures that can be folded into a rectangular package of stripes, similar to honeycomb cores of classical sandwich panels.
As assumed in earlier research on linear folded stripes, some rotational bodies seem to act as deployable folded structures, but whether other surfaces can be approximated or generated with linear folded stripes has not yet been investigated. Some first investigations on other translational surfaces are promising, but as the stripe generated system is a very complex mechanism, it could not yet be verified.

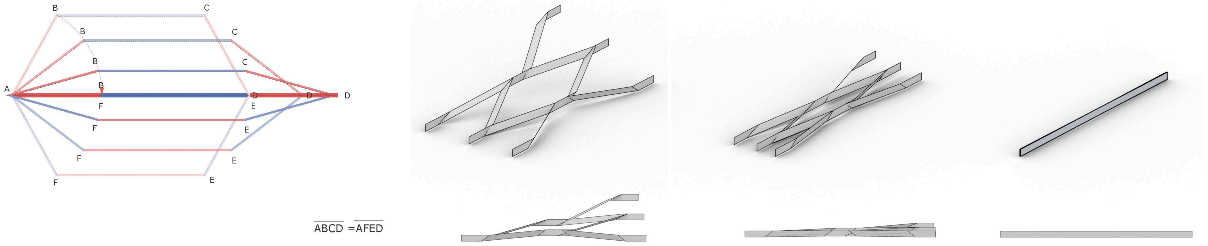

Fig 2 Equal segment length and convertibility with parallel linear folded stripes

A reticular system of linear folded parallel stripes consists of several stripes, wherein two stripes are necessary to form hexagonal cells. Each stripe is a rigid foldable mechanism. Until now most structures were generated for a three dimensional structure, with the aim that each stripe is a rigid, foldable system. For this particular research the deployability of an entire system will be investigated. There is an assumption that assembled structures can, under certain circumstances, be folded into a flat rectangular package.
It can be assumed that, even if linear stripe segments are rigid in both their unrolled and final three-dimensional assembled state, the motion they exhibit to transform from one configuration to another could utilize non-rigid folding transformations.

Some algorithms will be presented, that enable equal sided hexagonal stripe structures, and their behaviour in assembled state to be analysed. This paper will present current theories and first results of this research.

Self-organized Folding with Gravity and Friction as Guiding Concept

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This paper represents the on-going research on self-organized folding with gravity and friction as guiding concept. The aim of this research is to find a method to increase the predictability of the folding behavior of textile concrete, and constrain this behavior to existing folding theories. Even if there is a lot of research in the field of textile concrete and its material properties, there is no research in the field of its textile behavior within the production process and its use for design generation. As the textile concrete is produced in sheets, that have interesting textile properties, the authors got inspired to use this material in order to create formed sheets.

Keywords: Textile Folding, Fibre Concrete, Material Properties, Self-organizing system

First prototypes lead to the assumption that textile behavior of fibre concrete enables a folding technique that is influenced enormously by its boundary conditions. The two boundary conditions for the self-organized folding are gravity and friction. The material properties can be defined as a constant factor.

Gravity is an important factor, as the self-weight of the wet sheets that are produced are very difficult to handle, and even more difficult to simulate from scratch.

Friction as the second main factor, which can be defined throughout the process, is defined by the friction properties of the table, where the material is manipulated on. As the weight and the stiffness properties of the wet material are forcing the production process to a table, the material in its initial state is in full contact to a flat ground.

However these constraints in combination with the knowledge of classical folding properties will lead to a new formal language for self-organized folded fibre concrete plates.

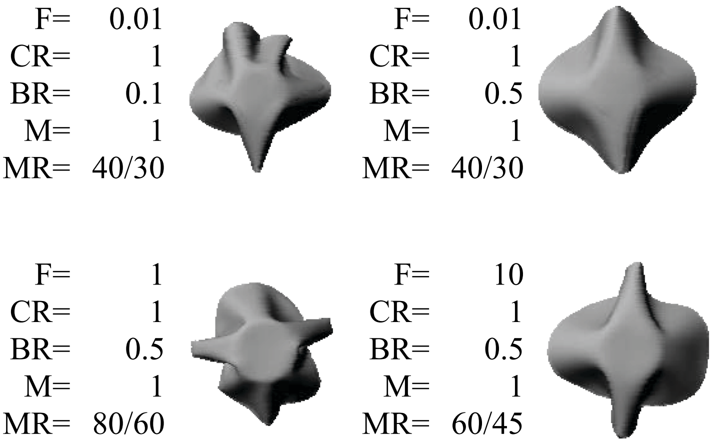

Fig. 1 digital simulation of self-organized folding with various parameter: Friction(F), Compression Resistance(CR), Bend Resistance(BR), Mass(M), Mesh Resolution(MR)

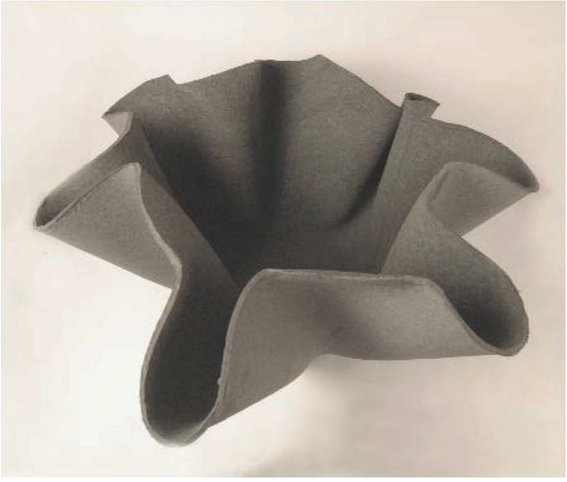

Fig. 2 self-organized folded prototype made from fibre cement

Folding Polygons to Deployable Convex Polyhedrons

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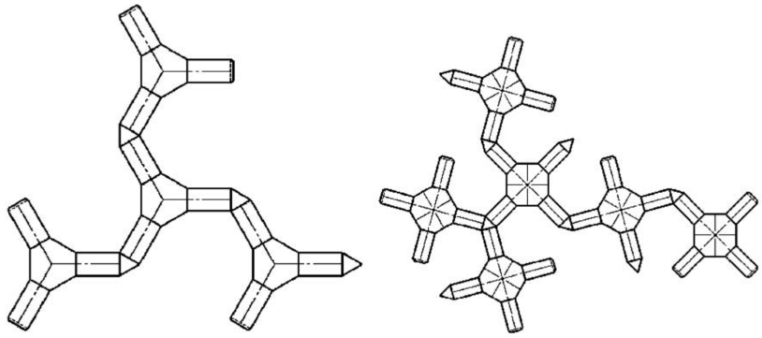
Abstract

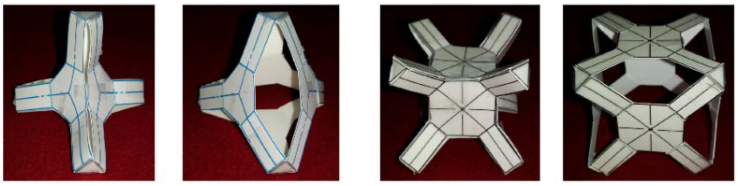
Origami has a close relationship with mechanism design. Inspired by origami folds, novel mechanisms possessing the feature of flat-foldability have been discovered and invented [1]. In reverse, by treading the links and joints in a mechanism respectively as panels and creases, new origami folds and crease patterns can be generated. Extending the Dürer's nets [2] for polyhedrons to a family of deployable polyhedral mechanisms [3], this paper presents new nets for folding a group of origami deployable convex polyhedrons (see Fig. 1 for two examples). Comparing to the unfolding and folding of traditional refold-rigid polyhedra [3], all the polyhedrons (including all the regular and semi-regular polyhedrons) proposed in this paper, which are folded from planar pieces of cardboards, have one mobility and are able to perform radially reciprocating motion.

Graph theory and matrix representation method are employed to describe and identify the process of folding the nets to their corresponding deployable polyhedrons. A new way is developed for modelling topological changes of folding the origami folds with the introduction of an EU-elementary matrix operation to produce the configuration transformation. Further, movability and mobility of the proposed deployable polyhedrons are analysed and verified using symmetry-extended mobility rule together with screw-loop equation method. Then, the radially reciprocating motion of the polyhedral folds is formulated and characterized with examples and simulations.

This paper hence not only provides insight into construction of deployable cardboard origami folds, but also suggests new approaches for describing the unfold-refold process of manipulating origami folds from their unfolding polygonal pieces.



1a) Net for a deployable tetrahedron 2a) Net for a deployable hexahedron



1b) A deployable tetrahedron and its two phases 2b) A deployable hexahedron and its two phases

Fig. 1 Nets and their corresponding deployable polyhedrons: two examples

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Sound Insulating Performance on Origami-based Sandwich Trusscore Panels

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Abstract

In this presentation, the sound insulating performance of high-rigid sandwich panels called “trusscore panels” is numerically evaluated by an acoustic finite element method. The trusscore panel (Fig. 1) is designed based on a spatial tessellation technique filling space by polyhedrons and manufactured by pressing metallic flat plates and welding the pressed plates [1].

Theory: Transmission loss (hereafter, TL) is a criterion to evaluate the sound insulation performance. The TL for a single flat plate is formulated as

$$TL = 20\log_{10} mf + 10\log\left[1-\left(\frac{f_r}{f}\right)^2\right]^2 - 42.5\,, \qquad (1)$$

where f is a frequency, m is the surface density of the plate, and f_r is a resonant frequency of the plate given by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}}\,. \qquad (2)$$

Similarly, the TL for double-leaf plates is formulated as

$$TL \;=\; 10\log_{10}\left[1+4\left(\frac{\omega m}{2\rho c}\right)^2\left\{\cos\frac{\omega d}{c}-\left(\frac{\omega m}{2\rho c}\right)\sin\frac{\omega d}{c}\right\}^2\right] + 10\log\left[1-\left(\frac{f_r}{f}\right)^2\right], \qquad (3)$$

where d is the distance between the double-leaf plates, ρ is the density of the plate, c is the sound velocity, and $\omega(=2\pi f)$ is the angular frequency.

Numerical results: The trusscore panel is simply supported at the edges and placed between two chambers (Fig. 1). The sound pressures p_j at every node in the chambers are computed and summed up to obtain the TL by the following equation.

$$TL = 10\log_{10}\frac{E_{source}}{E_{received}}\,, \text{ where } E = \frac{\frac{1}{2}\rho\sum_{j=1}^{N}\frac{{p_j}^2}{\rho^2c^2}V_j}{\sum_{j=1}^{N}V_j}\,. \qquad (4)$$

Figure 2 shows the computed TL of the trusscore panel and the theoretical TLs of the corresponding single plate and double-leaf plates given by Eq. (1) and (3), respectively. It demonstrates that the trusscore panel can insulate sounds effectively at low-frequency range; 0 – 400 Hz in this example. Note that the range depends on the first natural frequency of the panel. Furthermore, the behavior is exactly the one for the corresponding single plate. Even if the trusscore panel has hollows between the plates, the property of the TL does not follow the one for the double-leaf plates. We conclude that the trusscore panel can insulate low-frequency noise effectively according to the high rigidity and that the hollows do not affect the sound insulation, because the plates are firmly jointed, and therefore, the acoustic energy cannot be damped by air in the hollows.

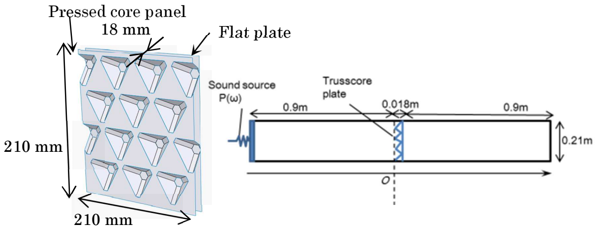

Fig.1 Dimension and setup of the trusscore panel

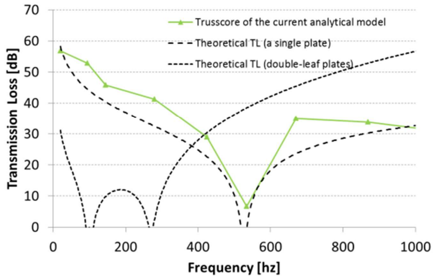

Fig. 2 Transmission loss of the trusscore panel; the theoretical TLs for the corresponding single plate and double-leaf plates are given, assuming the identical mass to the trusscore panel.

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MAGNETIC SELF-ASSEMBLY FOR THREE-DIMENSIONAL MICROSTRUCTURES

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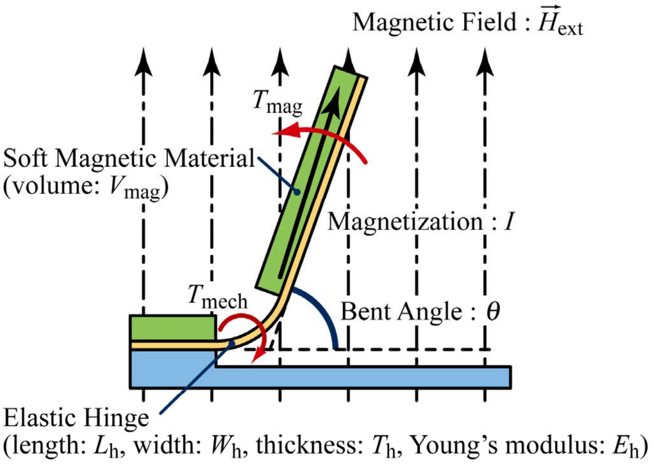
iwase@iwaselab.amech.waseda.ac.jp

We have developed a three-dimensional (3-D) microstructure assembly by an external magnetic field. In a small world, a magnetic field perpendicular to a substrate lifts up a soft magnetic micro-plate due to the shape magnetic anisotropy of micro-plate. This method can assemble 3-D microstructures from 2-D structures similar to “Origami (paper folding)”.

Fig. 1 shows a principle of our method. When a thin micro-plate of soft magnetic material like Permalloy is placed in a magnetic field, magnetic torque is generated depending on the shape anisotropy. This is the same reason as iron sands stand up on a permanent magnet. Our basic structure consists on a rigid magnetic micro-plate and elastic hinges as shown in Fig. 1. By theoretical calculation of a magnetic and a mechanical torques, we derived a dimensionless factor which determines the sensitivity of the hinged micro-plate to a magnetic field. By using this factor, we can design the assembly order of each plate. Therefore, we can achieve multi-step sequential assembly and complex 3-D assembly. When we set appropriate differences in sensitivity to each plate, 3-D structures can be assembled by only placing the structures on a permanent magnet.

We have fabricated hinged micro-plates, which consist of 4.5-μm-thick electroplated Permalloy plates and 200-nm-thick nickel elastic hinges of various sizes. Typical micro-plate size is several hundred μm (from 200 μm to 800 μm). We have achieved four-step sequential assembly of four micro-plates and assembly of complex 3-D microstructures with closed linkage. Fig. 2 shows assembled push-up linkage structures as magnetic actuators. The top plates can move up and down according to a strength of magnetic field.



$$T_{mag} = V_{mag} |\vec{I} \times \vec{H}_{ext}| = V_{mag} I H_{ext} \cos\theta$$

$$T_{mech} = E_h \frac{W_h \cdot T_h^3}{12 L_h} \theta$$

Fig. 1 Direction and magnitude of torques acting on hinged magnetic plate in a magnetic field.

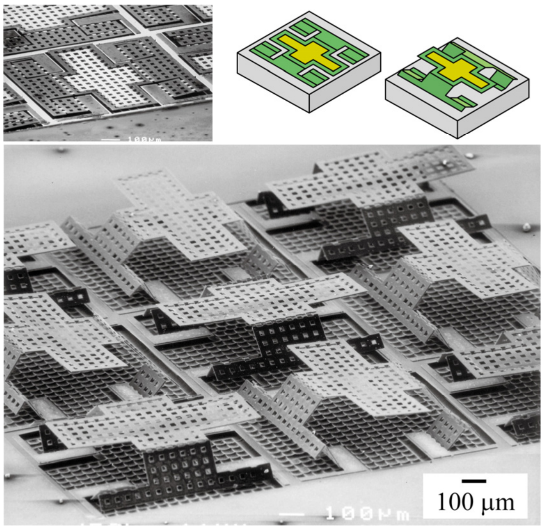


Fig. 2 3-D microstructures as magnetic actuators.

Shovel Folding: Algorithmic Origami Design of Words and Other Line Drawings

Wensdy Whitehead

Shovel folding techniques provide a systematic approach to box-pleating to allow quick 3D origami design of line drawings without massive collapses or even an Elias stretch. It is a family of related origami techniques that work very powerfully together. It includes basic shovel folds, inverse shovel folds, half shovel folds, half inverse shovel folds, sliding shovel folds, hesitating shovel folds, null shovel folds and a "corner-turning" procedure. Inverse techniques create the same structures as their non-inverse counterparts but in reverse, causing the working end of the developing model to become narrower rather than wider. The various types of shovel folds work together in sequence across pleated paper, generating three-dimensional structures seen edge on, rather like a line drawing with three-dimensional ink hanging in mid air. This sequential nature allows a very algorithmic approach to design, including subroutines for all the capital letters of the English alphabet. These shovel-folding subroutines provide enough power that the total design time for eight new words for an exhibit was under five minutes and the first execution of all eight designs were display quality. This paper will address the underlying shovel folds, however, rather than specific subroutines built upon them.

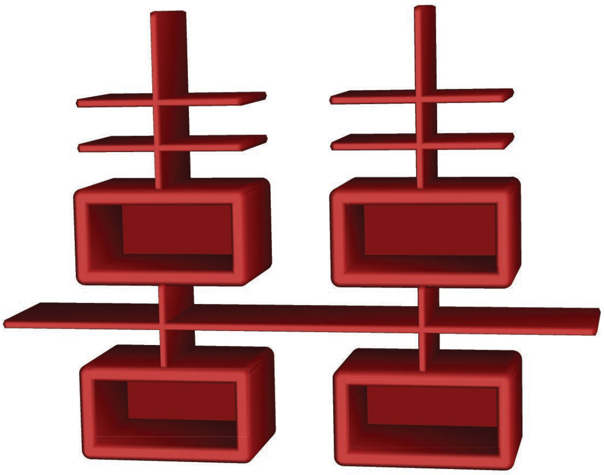
Most shovel folding designs use a single rectangle of paper, but the technique also applies to modular design.

The techniques of shovel folding are not inordinately difficult. Intermediate folders with precision and patience can handle the various folding techniques required, so most models developed under this method will be intermediate or high intermediate.

Shovel folding starts on a square grid – *any* square grid. Powers of two (2) are particularly handy as grid dimensions, so the explanations will use 1:2 paper with an 8x16 grid to illustrate the techniques.

Shovel folding gets its name from the appearance of the basic shovel fold halfway through execution.

A shovel-folding design from 1:2 (8x16 grid).



A two-piece shovel-folding design.

Curved-Folding Convex Polyhedra through Smoothing

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The research documented in this paper operates on the backdrop of the exciting potentials that the field of curved crease folding offers both in development of curved surfaces that can be fabricated from sheet material and also in their structural performance. Specifically, it presents a geometric method of curved-folding smoothed, convex polyhedrons from flat sheets and a computational framework to design with and digitally reconstruct such surfaces (Figure 1). This method could find applications in architectural and industrial design as it describes a cost-effective and relatively simple method of building specific kinds of curved surfaces that are also structurally rigid due to the closed polyhedra they are derived from.

As a research prototype, this paper also documents the design and fabrication of a human-sized installation that was built out of sheet aluminium using this method (Figure 2). The installation was the conclusion of a ten-day architectural workshop on curved folding held in Bangalore, India in November 2012. During the construction process only the cutting of the sheets required a laser cutter; all of the folding and assembly was done by students with their hand in less than a day – proving to demonstrate the simplicity in fabrication of the method described.

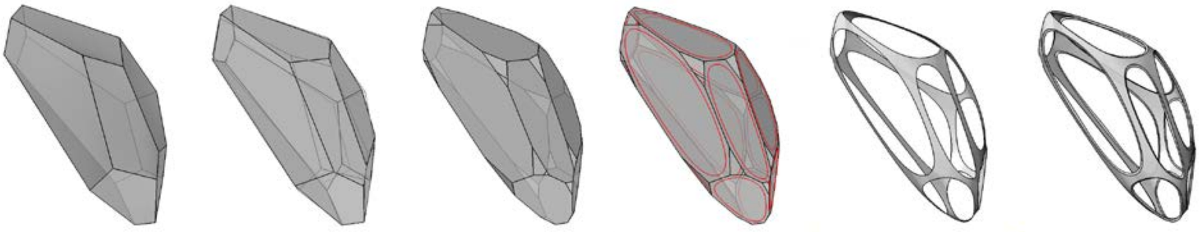


Figure 1: *The Geometric Method from Left to Right: Convex Polygon, Face Planarization, Vertex Chamfering, Face Smoothing, Resurfacing, Boundary Extruding Inwards*

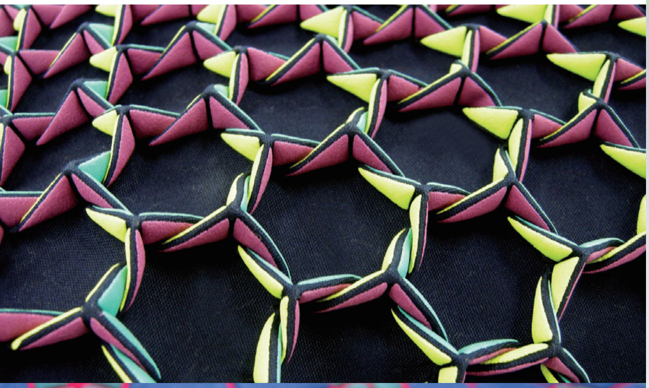
Figure 2: *The finished prototype installation made from 1mm thick sheet aluminium*

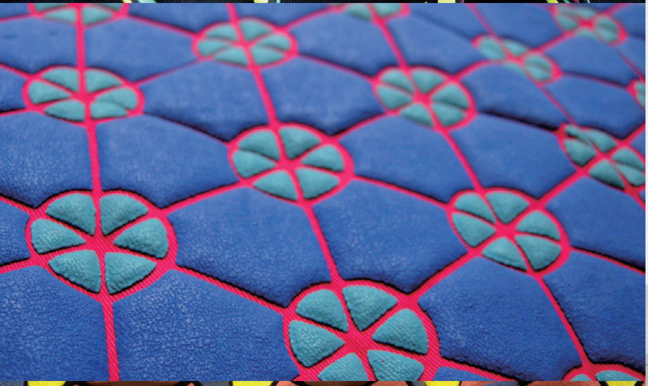
6OSME Categories: art, design, technology
Keywords: architectural geometry, curved folding, computational framework for object modelling

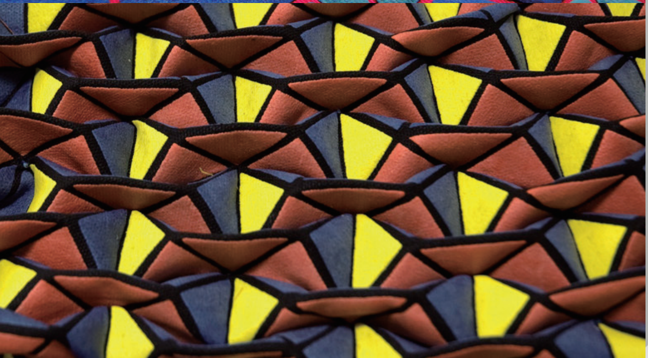
POP-UP PATTERN

Maori Kimura

I made use of the characteristic of puff printing that the printed cloth becomes hard, and propose fabrics that can be folded along the form of the pattern.
Using Ron Resch patterns and original patterns, I made three-dimensional designs that seem to pop up of the fabric.
I expressed various forms deriving from a single pattern in the form of bags.
These bags are easy to carry as they are stretchy and shock-absorbing.







paper_165

A modular collapsible folded paper tower

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Compliant and deployable structures are of particular interest to the aerospace and medical device industries, where traditional structural designs require integration of complex mechanisms to enable reconfiguration, often leading to compromises in other performance attributes such as weight and cost. One way to enable large-scale shape changes in compliant structures is to use folding and buckling of the constituent materials. As a demonstration study using Origami-inspired principles, we present a modular collapsible paper tower exhibiting unusual mechanical behavior (Fig. 1a-c), including reversible buckling of its segments.

To fabricate the tower, individual unit cells ("storeys") are layered in series; each storey is a folded heptagon (Fig. 1d) laser-cut from a sheet of paper, with perforations along the folds. The cells are folded and then glued together, and can collapse independently when the tower is compressed. When collapsed and compressed completely, each has rotated by 77^{o} relative to its fully extended configuration.

We tested the paper tower in compression. A multi-storey tower has periodic force-displacement curve (Fig. 1c). A stress-strain plot of a 2-storey tower with rotation constrained is shown in Fig. 1e. The fully extended configuration of the 2-storey tower is defined as 0% strain, and the fully collapsed configuration is at ~90% strain. Within this range, we find that the mechanical behavior has four regimes. In Regime I, the structure shows roughly linear force-displacement behavior as it is compressed, until a peak force is reached and the sidewalls of the cells buckle. In Regime II, as the structure contracts, we observe a negative effective modulus of ~5kPa, calculated based on the heptagonal cross-section. Past the minimum in the force-displacement curve (Regime III) the stiffness is dominated by flexure of the perforated creases, and we observe a positive effective modulus of ~1kPa. Finally, at strain exceeding ~85%, the segments of the sidewalls contact and the stiffness increases significantly.

We conclude that the mechanical behavior of the collapsible tower arises from its geometry in combination with the kinematics of folds, which guide the structure through a trajectory causing the observed instability. Future work will explore the dynamic properties of the design and its fabrication out of composite materials towards useful macroscopic structural behavior.

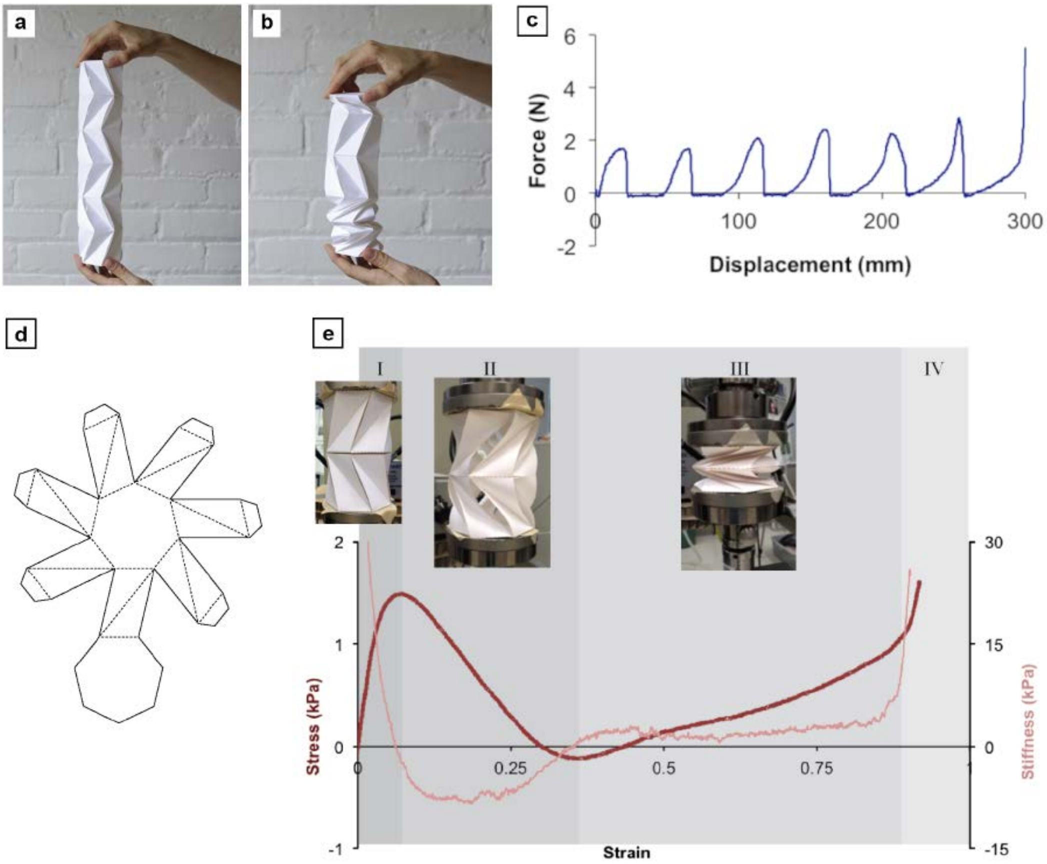

Figure 1. Design and mechanical behavior of the paper tower. A 6-storey tower (a,b) has periodic force-displacement behavior, and without constraining rotation each cell collapses independently (c). The pattern of of heptagonal folded unit cells is illustrated in (d). A 2-storey tower with rotation constrained has (e) unusual mechanical behavior and a negative stiffness after buckling of the sidewalls (e).

Millimeter scale self-folding of a laminated paper system

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Macroscale self-folding systems have been demonstrated using a variety of active materials along with optical[1,2], electrical[3], and thermal actuation[4]. However, there has been little exploration of folding methods at intermediate scales between the limits of microfabrication and manual manipulation. Miniaturization of self-folding processes toward millimeter and sub-millimeter scales requires an in-depth understanding of fold mechanics via constitutive relationships. Moreover, folding of sheet materials at millimeter and smaller scales is challenging due to the mechanical limitations on the fold radius, the accuracy and roughness of cutting and scoring methods and tools, and the need to apply localized forces. Our goal is to identify the dimensional limits to self-folding of conventional Origami papers, via analysis along with experiments using patterning and lamination of active layers onto the paper.

We begin by modeling the mechanics of an exemplary system, comprising a paper substrate of thickness $2t$ and fold length w (Fig. 1a). The folding system is modeled using an orthotropic material in pure bending with elastic-perfectly plastic behavior. Using a plane strain simplification[5], the following moment-curvature relationship is derived. Here, M_e and M_p are the bending moments in the elastic and plastic regimes, respectively, κ is the curvature of the fold, and σ_0 is the yield strength. The critical curvature κ_c corresponds to the formation of a plastic hinge, i.e., the onset of plasticity.

$$M_e = \frac{2E_x t^3 w}{3\left(1-\nu_{xy}\nu_{yx}\right)}\kappa \qquad\qquad M_p = \frac{\sigma_0 t^2 w}{\left(1-\nu_{xy}\right)}\left[1-\frac{1}{3}\left(\frac{\kappa_c}{\kappa}\right)^2\right]$$

$$\kappa_c = \frac{\sigma_0\left(1-\nu_{xy}\nu_{yx}\right)}{E_x t\left(1-\nu_{xy}\right)}$$

To demonstrate this approach, we have designed and fabricated centimeter-scale self-folding hinges using standard copy paper (97 µm thick). The paper is cut and scored using a mechanical plotter cutter (Graphtec FC4200) or a laser cutter (60W, 1% power setting). We then fabricate hinge assemblies by laminating a heat-sensitive shape memory polymer (Basket Accent 75 gage Polypropylene) and a graphite heat distributor (Panasonic Pyrolytic Graphite Sheet) onto the paper using thin adhesive (Adhesives Research ARClad 8206). The graphite sheet is heated electrically and it distributes heat to the polypropylene, activating the polymer and reaching a final plastic curvature of 2800 m^{-1}.

To inform the model, the thermo-mechanical behavior of the active polymer, and the shear and peeling strength of the silicone adhesive are measured (TA Instruments RSA3), while the elastic constants of paper are obtained from previous studies.[6,7] As shown in Fig 1d, due to the formation of the plastic hinge the required actuation moment required asymptotically approaches a limiting value for increasing curvature, within the limits of non-ideal plasticity, fracture stress, and delamination. Furthermore, the actuation moment required reduces significantly with substrate thickness. The model therefore predicts that the moment applied to the prototype hinge is approximately 0.006 N/m. In general this model enables selection of the active material and actuation mechanism for self-folding, and could be extended to analyze other primitive fold elements such as mountain-valley folds, and later to sheets involving fold interactions.

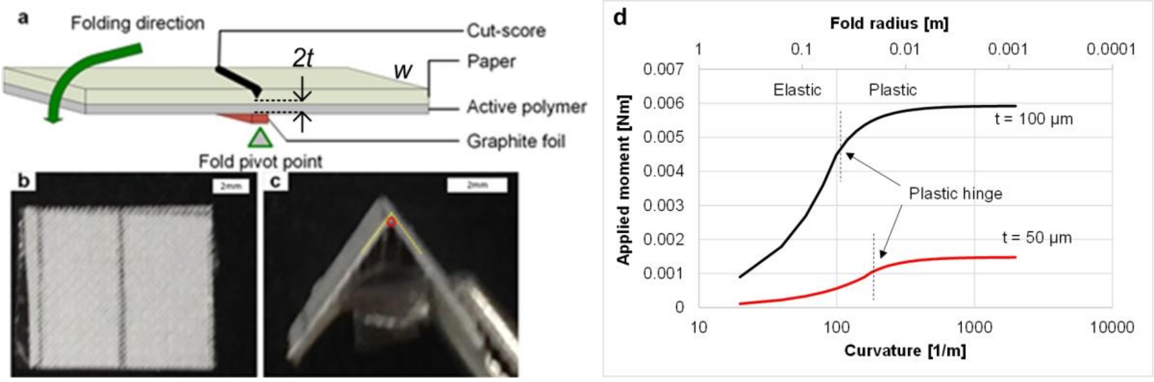

Figure 1. Self-folding model and experiment: (a) schematic of laminated and patterned stack; (b, c) simple hinge before and after electrically activated folding; (d) Folding moment versus curvature, showing elastic and plastic regimes.

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Designing Curved-Crease Tessellations of Lenses: Qualitative Properties of Rulings

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Martin L. Demaine
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David A. Huffman
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Duks Koschitz
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Tomohiro Tachi
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Abstract:

We describe a general family of curved-crease folding tessellations consisting of a repeating "lens" motif formed by two convex curved arcs. The third author invented the first such design in 1992, when he made both a sketch of the crease pattern and a vinyl model (pictured below). Curve fitting suggests that this initial design used circular arcs. We show that in fact the curve can be chosen to be any symmetric convex curve without inflection point. We identify the ruling configuration through qualitative properties that a curved folding satisfies, and prove that the folded form exists with no additional creases, through the use of differential geometry.

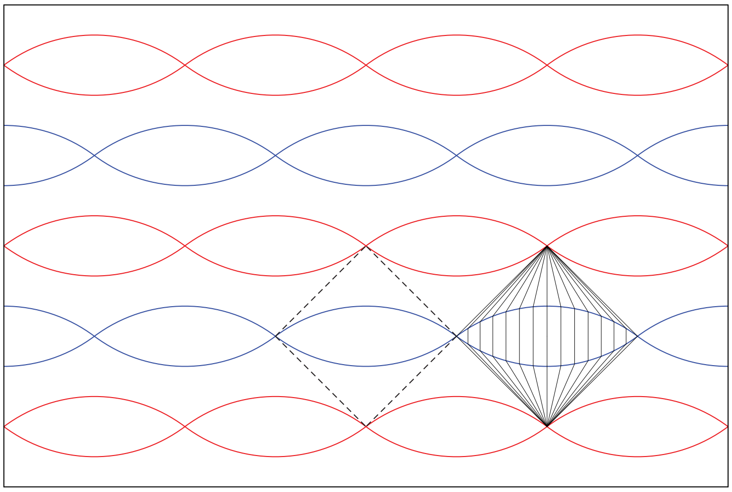
The 3D configuration of the curved folding is solved through identifying the correspondence between pairs of points connected by rule lines. Here, we derive two qualitative properties from the differential geometry around a 3D crease curve and between the curves:

1. A smooth 3D crease cannot have incident cone rulings.
2. Two concave curves cannot be joined if their mountain/valley assignments differ.

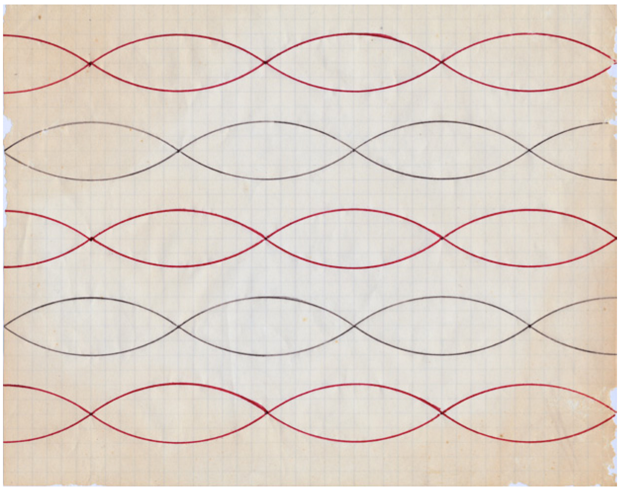
These properties separate the tessellation into independent kite-shaped tiles and force the rulings between the lenses to be particular cones with their apices coinciding with the vertices of the tiling. The ruling inside each lens is free (can twist), but assuming no twist or global planarity/symmetry, is cylindrical (vertical rule lines). The tiling exists by rotation/reflection of the 3D model of each kite around its four straight boundary edges. From the tiling symmetry, each tile edge has a common tangent to its neighbors regardless of the type of curves, as long as it is a convex curve. The curved-crease model is also "rigidly foldable", meaning that it can be continuously folded without changing the ruling pattern.

Category: Mathematics
Keywords: curved creases, tessellations, rule lines

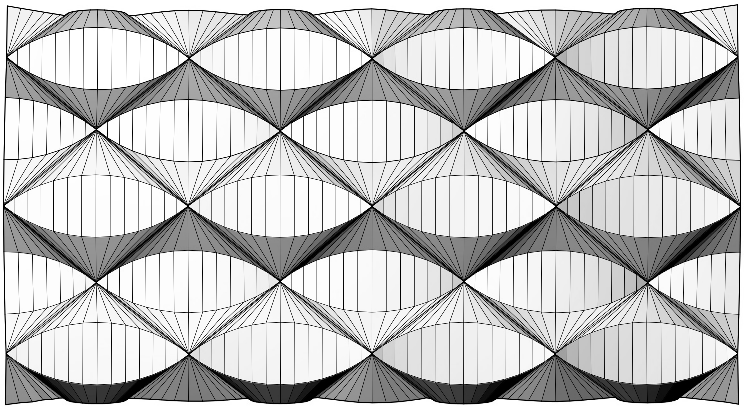
Acknowledgments. We thank the Huffman family for access to the third author's work, and permission to continue in his name.



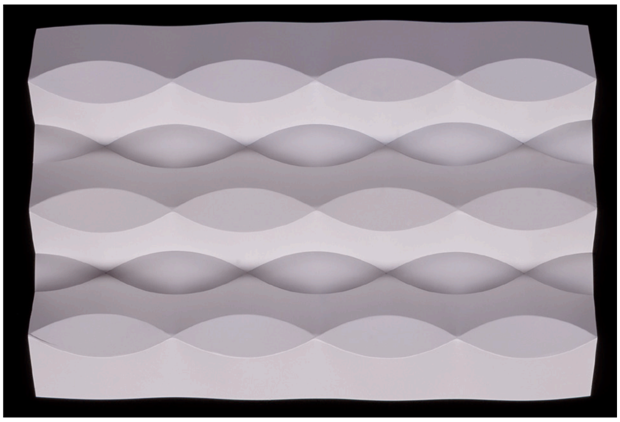
Digital crease pattern of lens design



Original crease pattern sketch by third author (1992)



Simulated 3D model using Freeform Origami software



Original vinyl model by third author (1992)
[photo by Tony Grant]

„The next model is for Beginners“
First steps in *Differential Origami*

Michael J. Winckler[1]

Abstract

When folding origami with beginner classes, the selection of models largely influences the success of the class and the fun of the participants. Model designers have a rather good idea about the difficulty of the folding process and usually classify them in categories like „beginner“, „advanced”, „difficult“. Along the same lines, experienced teacher have a good feeling what models they can present to their class, choosing from a somewhat larger personal classification for the difficulty level of models.

Such a classification is not really based on one strikt criterion: Some models are more advanced because they have areas with a larger number of layers, some other models are more advanced because they use more difficult folding operations and some models are more advanced, because the precision on single folding operations must be high to ensure the quality of the final result. It is the latter characteristics that we are interested in for this research.

Defining a flat origami model by its folding algorithm, the exact sequence of folding operations is the key ingredient to analyze the *precision complexity.* The question that we directly address is: How much does the position of final parts of the model depend on the individual folding steps? In this context, the final parts are usually identified by points of interest such as the tip of a wing or a specific corner.

This approach is similar to the computation of variations of a simulation result with respect to specific input parameters: We start out with a parametrization of the complete folding sequence. When actually folding the model, these parameters (such as angles or postions) will be performed by the folder with a certain precision, which can be represented by a statistical variation in the respective parameter. This variation will thus represent the *local error* of a specific step.

Due to the topolgy of the folding, some local errors get augmented during further folding steps, while others actually get reduced. This behaviour can be modeled by an analysis of the *error propagation*, which describes the development of a local error during further steps of the folding sequence.

This analytical method of model complexity should be combined with actual experimental work: Local errors for specific folding operations greatly depend on the experience of the folder, the clarity of the description of the folding operation and several other factors, which are even more difficult to assess e.g. paper quality.

The combination of an analytical method with statistical empirical data makes this analysis method especially interessting and rewarding.

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Determining Distinct Shapes of Rigid Origami

Zhonghua Xi and Jyh-Ming Lien*

Abstract

Rigid origami is a class of origami whose entire surface remains rigid during folding except at crease lines. In this work, we explore the idea of determining distinct shapes that can be realized by a given crease pattern. Typically, crease pattern is designed with a single target shape in mind. However, as the advances in material science and robotics engineering enable the realization of self-folding rigid origami [Ryu et al. 2012; Ahmed et al. 2013; An et al. 2011], it is desirable to create a crease pattern that can fold into multiple shapes. Traditionally such a crease pattern is created by combining the crease patterns designed for each of the target shapes. The main limitation of such an approach is the complexity of combined crease, which often prohibits the implementation of self-folding origami due to the geometric, kinematic and material constraints. Therefore, we propose to take an approach that attempts to take an existing crease pattern and discover most distinct k foldable shapes. Our hope is that we can match one of the k distinct foldable shapes to the given target shapes, and only modify the crease pattern locally to better approximate the target shapes.

We say that two foldable shapes (origami) s and s' are distinct if their folding paths from a flat sheet are in different homotopy classes and the distance between s and s' is greater than a user defined value. Our method proceeds by creating a random foldable shape s and keeping s if s is distinct from the existing distinct shapes S. This process iterates until S has k elements. More specifically, we model the rigid origami as a kinematic system with closure constants and solve the foldability problem through a randomized method and determine the distinctness of shapes by careful foldability analysis. Fig. 1 illustrates this idea. Our experimental results show that the proposed method successfully determines distinct shapes from several types of rigid origamis.

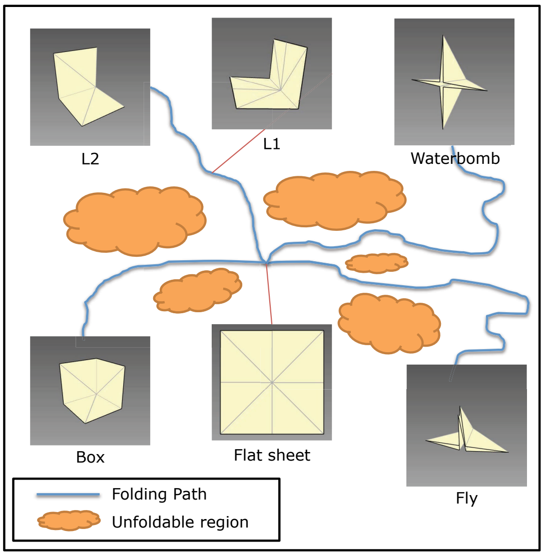

Figure 1: *This figure illustrates the ideas of determining the distinct shapes from a given crease pattern. Each folding path in this figure is obtained from different homotopy classes.*

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Weaving a Uniformly Thick Sheet from Rectangles

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Any child knows how to weave a few strips of paper into a sheet, which happens to be of uniform thickness. However, the size of the sheet in this simple weaving is limited by the length of the strips used in creation. This sheet also requires some sort of external locking mechanism to hold it together. Without tape or a non-uniform edge, this sheet will slide apart.

By contrast, we show how to weave together finite-length strips into an infinite sheet of uniform thickness. In addition, our sheet is locked, and will not slip. However, our weaving requires more layers than the child's model. For an arbitrary rectangular strip, the sheet is, at worst, eight layers thick. For an arbitrary polygonal "strip", the sheet is no thicker than eighteen layers. In the worst case, we conjecture that these are also the lower bounds. In some special cases, we show that the sheet can be thinner; for example, using 1×5 rectangles, we can make a sheet that is just five layers thick (see Figure 1). It is also possible to reduce to four layers of thickness with carefully designed concave polygons. This paper proves our upper bounds on thickness, and gives general sheet-weaving algorithms.

While creating a locked, infinite sheet of uniform thickness proves to be relatively simple, creating a finite sheet is far more complex. In a finite sheet, boundary conditions come into play. If we just apply a portion of our infinite constructions, the edges of the sheet are ragged, and locking the sheet while keeping the thickness constant proves tricky. We describe solutions to these problems, resulting in locked, uniform-thickness, finite woven sheets, but at the price of further increased thickness.

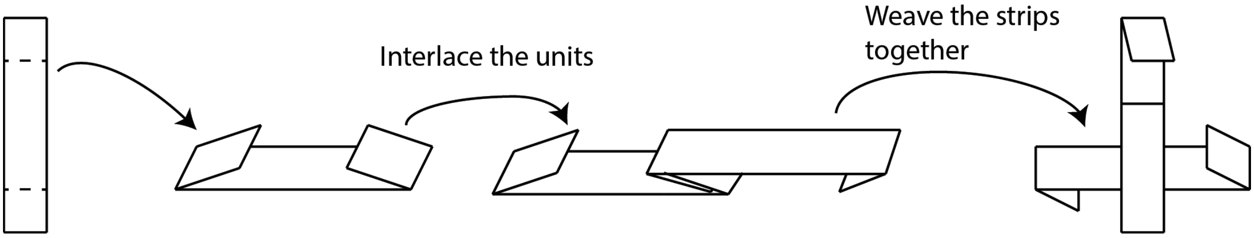


Figure 1: Creating a 5-layer woven sheet from a 1-by-5 rectangle.

Origami and the Language of Design

Peter Engel
Architect and Origami Artist, Berkeley, CA USA

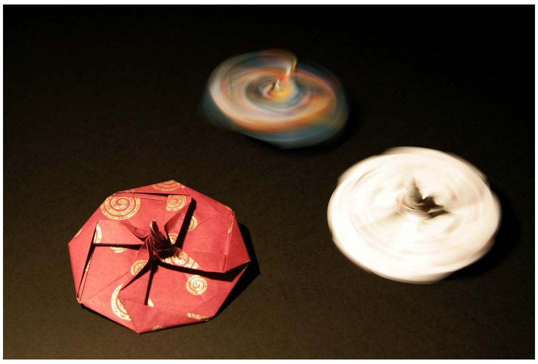
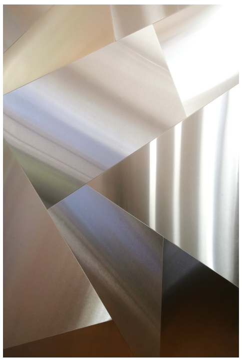


Visual and experiential art forms such as painting, sculpture, architecture, music, and dance draw on universal design principles. The artists who work in these varied media employ design principles such as form, rhythm, proportion, and color to create their artistic products.

Unlike these more established art forms, however, origami has never developed a vocabulary of design principles to aid in creating and critiquing new models as works of art and craft. In the opinion of American origami artist and architect Peter Engel, folders are quick to seize upon the representational aspect of an origami figure even though the folded paper model is not, by itself, a reindeer or a flower, no more than the rocks in a Zen garden are mountains or the notes in Debussy's *La Mer* the actual sounds of the sea. Rather, like a Zen garden, an origami model is an abstract composition of forms in space, perceived by means of light and shadow, and imbued with meaning and emotion. The composition triggers sensations and feelings in the viewer that have been shaped by that individual's own experience and our collective unconscious as a species.

In his presentation, Engel will make the case that as an abstract composition of forms in space, an origami model is subject to fundamental design principles such as symmetry and asymmetry, solid and void, form and function, pattern and rhythm, and scale and proportion. These principles are the language of design. The origami designer who harnesses them with skill and intention produces a beautiful work of art that evokes an emotional and meaningful response in the viewer.

Engel will illustrate his presentation with numerous examples of his own origami and architectural work (such as those shown here) and demonstrate how in his life these disciplines have inspired and influenced each other.

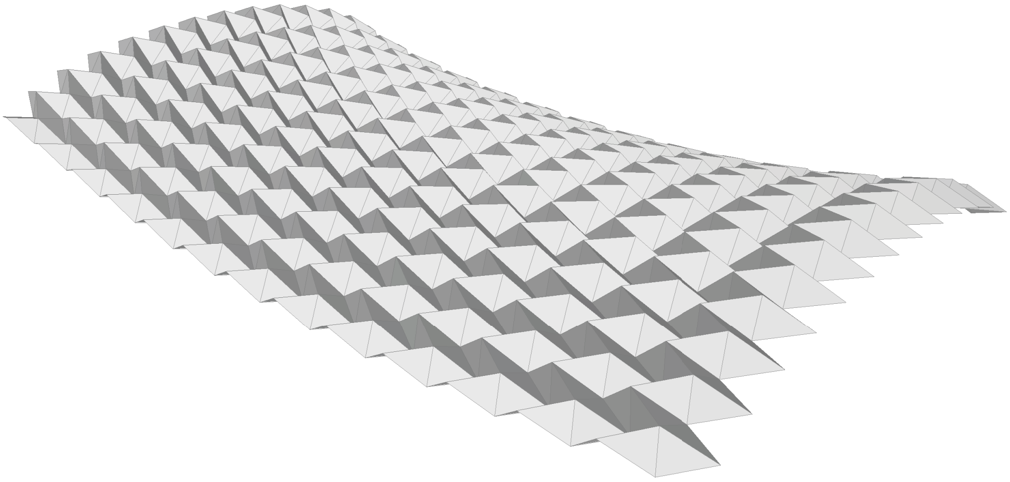




A Simple Mechanical Simulation of Curved and Tessellated Origami Structures

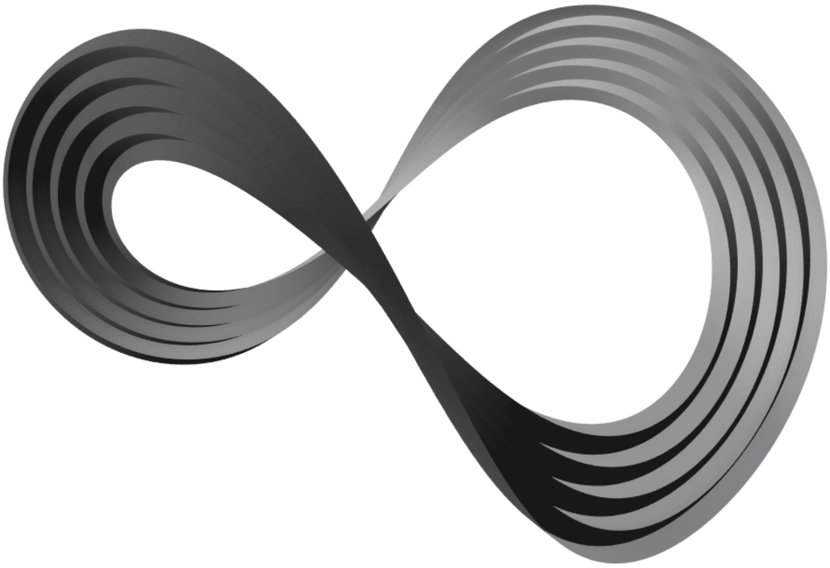
Levi H. Dudte, L. Mahadevan
Harvard University, Cambridge, MA, USA

Abstract

While the design of origami is rightly viewed as primarily a geometric exercise, many origami structures assume their form from the interplay of geometry with the mechanics of folded paper. We present a simple mechanical, computational tool for folding paper and motivate its creation by simulating origami structures with tessellated and curved folding patterns. In each of these examples the simulation tool becomes a method for investigating the mechanical properties of the origami structures. In particular, we show its utility in investigating the bending behavior of the Miura pattern, a tessellated folding pattern whose out-of-plane deformation modes can only access surfaces of negative Gaussian curvature, and the buckled shape of an origami structure with circular folds.



(a) Symmetric bending mode of the Miura Ori pattern



(b) Eight nested circular folds form a saddle shape

Project Mathigami: Exploring Mathematics through Origami

Authors: Rishika Daryanani, Celina Gonzalez, Paine Harris, Walshe Izumigawa, Bryce Lyon, Daniel Myers, Perla Myers, Tawni Paradise, Kerry Stanko, Veronica Verplancken, Anna Walsh, Elisabeth Yeruuldelger
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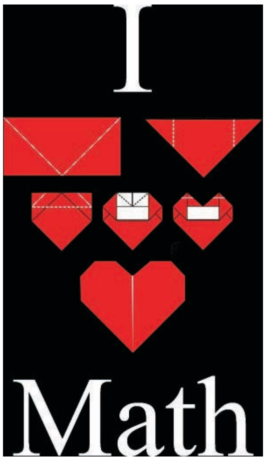
Students from the University of San Diego (USD) studying mathematics, computer science, engineering, neuroscience, art, and humanities, as well as future elementary and high school teachers, are building on other's work presented at past OSME conferences. In this pilot year of the program, the college student volunteers worked in teams to engage 3rd to 5th grade students in an underserved community to deepen the children's understanding of mathematics and create a positive attitude towards mathematics through origami. They also adjusted the lessons to teach younger children, middle-school students, and high school students. The lessons are focused on developing an interest in, and an understanding of, mathematics via the examination and exploration of mathematical concepts through the creation of origami models such as animals and interactive toys. The Mathigami project gives children, college students and future teachers positive experiences in mathematics and exposes them to the importance of mathematics in the development of origami and to the applications of both mathematics and origami to other areas.

The mathematical work includes developing an understanding of mathematical definitions, categorizing geometric objects, and exploring fractions, measurement, probability, scaling, etc. Mathigami aims to instill in students the belief that they can succeed in mathematics and the disposition to approach challenges with curiosity and perseverance; to develop critical thinking and problem-solving skills; and to improve their communication skills, spatial reasoning skills and geometric thinking skills.

This paper describes project Mathigami, and presents the results from this pilot year, lessons learned and future directions.



Design of the Origami-like Hinge Line of Space Deployable Structures

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Controlled one-way deployment of a large space structure is a key requirement_as there is no damping media in space._ The hinge lines of any space deployable structures can function of active hinge line where the one-way unfolding actuation is required but under controlled slow speed condition.

Fig. 1 shows our preliminary results predicting the hinge line opening during the deployment of plates connected with the hinge lines made of our proposed SMA/polymer composite. It is in space that the temperatures are extremely low, that such a cryogenic SMA must be used. An example of such a cryogenic SMA is FeMnAlNi whose stress-strain curves at various temperatures including cryogenic temperatures are shown in Fig. 2 which demonstrates relatively large superelastic (SE) strain of polycryalline FeMnAlNi. Another challenge in designing these hinge lines is that we would like the unfolding to occur across a wider temperature range rather than abruptly. The polymer (epoxy in our case) glass transition temperature will be altered to address this issue. This can be achieved by using two different epoxy materials with two different T_gs so as to create a new epoxy with diffused T_g.

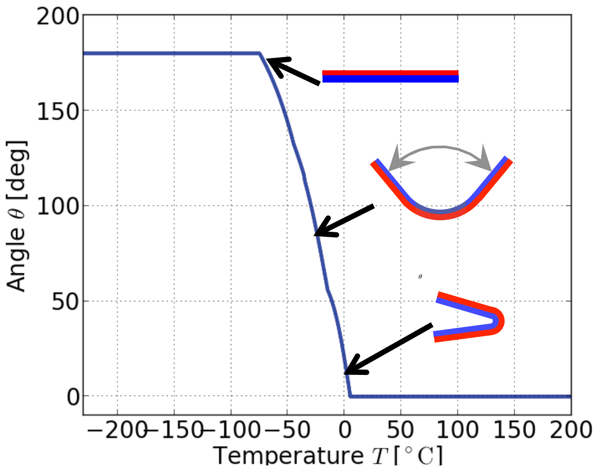

Fig. 1 Predicted history of angle vs. temperature of the SMA-modified epoxy hinge line where the epoxy is modified with diffusive Tg and the SMA is cryogenic SMA made of FeMnAlNi (Taya et al, 2013)

Another scheme of increasing the temperature range at which this transition occurs is to use the damping mechanism inherent in the superelasticity loop of the stress-strain curve of a cryogenic SMA, see Fig. 6. By using the above two schemes, we will be able to convert the otherwise rapid elastic energy release of the deployed space structure into a more controllable, gentle deployment.

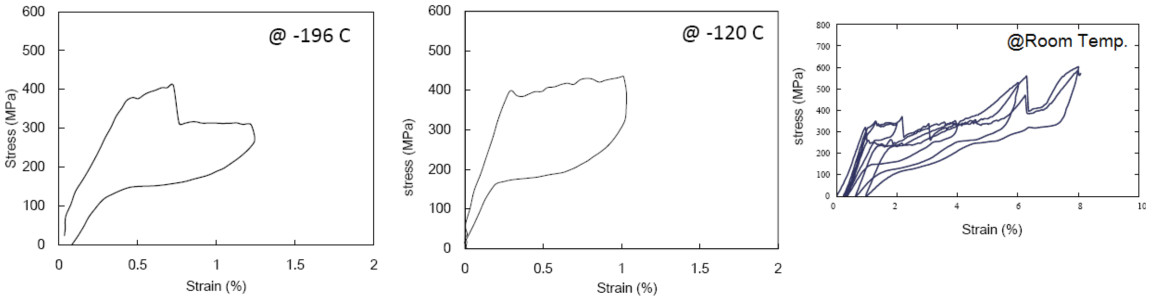

Fig. 2 Stress-strain curves of polycrystalline FeMnAlNi SMA tested at -196 °C, -120 °C and room temperature (Omori et al, 2011).

How to Fold an A4-paper Dividing into 101 Equal Length Parts without Any Measuring Tools.

Kazuo HAGA (Haga's Laboratory for Science Education)

Keywords: Odd-number division, Pendulum system, Three figures number

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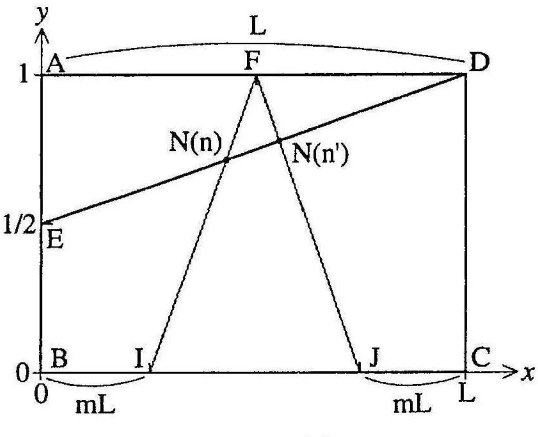
In Origami5 of the preceding OSME, I have showed a method to fold dividing lines on a rectangular sheet of paper into equal parts of an odd-number, both horizontally and vertically without any measuring tools .

The dividing method starts from a node making which is settled as an intersection of two straight lines. One of them is a constant line from the midpoint of the left side to the right upper corner of the sheet and is called a common line. The common line is suitable for trisection, 5-, 7-, 9-, 11-, 13-, 15-, 17- sections and go on. Another line starts from the midpoint of the upper side which is common and suitable for such odd-numbers and ends at a designated point on the bottom side according to a sort of the odd-number, and is called an individual line. To take an instance, when the individual line drops at the one fourth point from the left end of the bottom, we can get a 7-section node, while at the same length from the right end, we can get a 9-section node. A pair of these two individual lines looks like to swing left and right at the same width as a pendulum.

If the coordinates set on the rectangular sheet, bottom left corner as the origin, above-mentioned two straight lines can be express as two expressions, and the solution of simultaneous equations of them clearly indicates that those nodes are precise mathematically. This dividing method is applicable for all sort of rectangular paper including square one. I already did odd-number dividing on A4 industrial standard size paper from trisection to 31-section, both horizontally and vertically, and recently I have tried more larger number dividing. I accomplished 77-section three years ago and this year I am wrestling with a difficult assignment of the three figures number division as 101 of which individual line touches down on the 5/64 point from the left end of the bottom.



When BI is 5/64, N(n) is the node for dividing into 101 equal parts.

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Celina	Gonzalez	USA	Mission Valley YMCA P.R.Y.D.E. (Prevention, Recreation, Youth, Diversity, and Education)	179. Project Mathigami: Exploring Mathematics Through Origami
Georg	Grasser	Austria	University of Innsbruck, Faculty of Architecture Institute of experimental architecure, Innsbruck, Austria	158. Selforganised folding with gravity and friction as guiding concept

Marc	Grzeschik	Germany	Institut für Flugzeugbau / Universität Stuttgart	129. Comparison of Mechanical Properties of Periodic Non-flat Tessellations
Jordi	Guàrdia	Spain	Universitat Politècnica de Catalunya	123. Geometric and arithmetic relations concerning origami
Benedikt	Günther	Germany	Chair and Institute for Engineering Design, RWTH Aachen University	152. Comparison of Design Processes in Architecture and Mechanical Engineering
				153. Demands on an Adapted Design Process for Foldable Structures
Kazuo	Haga	Japan	Haga's Laboratory for Science Education	184. How to Fold an A4-paper Dividing into 101 Equal Length Parts without Any Measuring Tools
Ichiro	Hagiwara	Japan	Meiji University	161. Sound Insulating Performance on Origami-based Sandwich Trusscore Panels
				139. A study on crash energy absorption ability of lightweight structure by using truss core panel
				155. Origami-based Modeling Techniques for Deployable Meandering Tubes
				122. Approach for unfolding 3D meshes for crafting paper models
Paine	Harris	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
John	Hart	USA	MIT	166. A modular collapsible folded paper tower
				167. Millimeter scale self-folding of a laminated paper system
Darren	Hartl	USA	Aerospace Engineering Texas A&M in College Station	116. Miura-Ori Rectilinear Locomotion Using SMA Actuators
				94. Experimental Analysis of Self-Folding SMA-based Sheets for Origami Engineering
				48. Self-Folding of Reconfigurable Complex Structures using Programmable Active Laminates
Annette	Hatch	USA	University of New Mexico Valencia	133. Using the Fibonacci Series to Fold a Golden Rectangle
Koshiro	Hatori	Japan	Freelance	111. Saburo Murata and His Discovery of Maekawa's and Kawasaki's Theorems
				112. Nishikawa's Mitate Theory and its Relevance
Andrea	Hawksley	USA	Communications Design Group	19. Topological Origami Models of Non-Convex Polyhedra
Barry	Hayes	USA	--	85. Pentasia: an Aperiodic Origami Surface
Ryan	Hayward	USA	University of Massachusetts, Amherst	118. Geometrically controlled catastrophe and the link between snap buckling, origami, and material science
				31. Micro-patterned polymer gel multilayers for reversibly self-folding origami
Josef	Hegger	Germany	RWTH Aachen University	124. Oricreate: modeling framework for design and manufacturing of folded plate structures
Shinsuke	Hishitani	Japan	Hokkaido University	136. The effect of imagery controllability on Origami performance.
Akio	Hizume	Japan	Ryukoku University	63. Origami for triangular spiral tilings
Jonathan	Ho	UK	University of Oxford	33. Thin-Walled Deployable Grid Structures
Susanne	Hoffmann	Germany	Chair of Structures and Structural Design at RWTH Aachen University	125. A Systematical Overview of Origami-Based Structures in Technical Applications
				152. Comparison of Design Processes in Architecture and Mechanical Engineering
				153. Demands on an Adapted Design Process for Foldable Structures
Larry	Howell	USA	Brigham Young University - CMR Lab	140. Creating Novel Motions with N-Long Chains of Interconnected Spherical Mechanisms
				86. A Graphical Technique for Designing Rigidly Foldable Origami Mechanisms
				150. An Offset Construction Technique for Thick Rigidly Foldable Origami
Yanping	Huang	China	Southwest University. China	12. Using paper folding to solve problems in school geometry
Andrew	Hudson	USA	University of California, Davis	40. Using Origami to Enrich Mathematical Understanding of Fractals

David A.	Huffman	USA	University of California, Santa Cruz	168. Designing Curved-Crease Tessellations of Lenses: Qualitative Properties of Rulings
Thomas	Hull	USA	Western New England University	90. Locked Rigid Origami with Multiple Degrees of Freedom
				41. Counting Miura-ori Phantom Folds
				44. Rigid Flattening of Polyhedra with Slits
				76. Mechanics of Miura-ori Lattice Defects
				60. Symmetric Colorings of Polypolyhedra
				31. Micro-patterned polymer gel multilayers for reversibly self-folding origami
				77. Mechanics of Snap-Through Transitions in Twisted Origami
Tetsuo	Ida	Japan	University of Tsukuba	105. Automated Construction and Proving of Knot Fold by Eos System
Ushio	Ikegami	Japan	Junior high school teacher	55. Base Design of Snowflake Curve Model and its Difficulties
Fumihito	Imai	Japan	Hokkaido University	136. The effect of imagery controllability on Origami performance.
Sarah	Innes-Gold	USA	Tufts University	118. Geometrically controlled catastrophe and the link between snap buckling, origami, and material science
Sachiko	Ishida	Japan	Meiji University	161. Sound Insulating Performance on Origami-based Sandwich Trusscore Panels
				155. Origami-based Modeling Techniques for Deployable Meandering Tubes
Shozo	Ishihara	Japan	Saitama Prefectural University	70. Deformed Polyhedral Skeletons
Christopher	Itoh	USA	University of California, Berkeley	146. The Elusive Technique of Folding Anatomical Subjects
Jin-Ichi	Itoh	Japan	Kumamoto University	100. Flattening Polyhedra with Two Adjacent Rigid Faces
Eiji	Iwase	Japan	Waseda University	162. Magnetic Self-Assembly for Three-Dimensional Microstructures
Walshe	Izumigawa	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
James	Joo	USA	Air Force Research Laboratory	74. Physics-based Optimization of Origami Structures through FEM
Yoshihiro	Kanamori	Japan	University of Tsukuba	73. Simple Flat Origami Exploration System with Random Folds
				119. Designing Nth Order Rotational Symmetry Origami From 4th Order Symmetric Crease Patterns
				101. A Study on Orthogonal Pleat Tessellation and Its Folding Sequence
				96. Generating Folding Sequences from Crease Patterns of Flat-Foldable Origami
				72. Flat-Foldable Axisymmetric Structures with Open Edges
				97. Curved Origami for Developable Surface Coupling
Yuto	Kase	Japan	University of Tsukuba	72. Flat-Foldable Axisymmetric Structures with Open Edges
Takeshi	Kawakami	Japan	I.P. SYSTEM INC.	92. Origami on the iPad
Miyuki	Kawamura	Japan	Independent	128. Two Calculations for Geodesic Modular works.
Toshikazu	Kawasaki	Japan	Anan National College of Technology	104. On Quadratic Curves Generated from Quasi-Fish Bases
				107. A Meshod to fold generalized Bird bases
Maori	Kimura	Japan	Nexta Co.,Ltd.	165. [POP-UP PATTERN] Fabric of the ORIGAMI pattern to make with a puff print.
Anat	Klemer	Israel	National Center for Primary Mathematics Education, University of Haifa	66. Origametria: Research into its Effectiveness as a Tool for Developing Flexibility in Geometric Thinking
Yves	Klett	Germany	Institut für Flugzeugbau	131. Point of View: Joys and Uses of 3D Anamorphic Origami
				129. Comparison of Mechanical Properties of Periodic Non-flat Tessellations
Goran	Konjevod	USA	organicorigami.com	154. Origami beyond geometry
				142. On pleat rearrangements in pureland tessellations
				148. Characterization of Origami Corrugation Crease Patterns
Duks	Koschitz	USA	Pratt Institute	168. Designing Curved-Crease Tessellations of Lenses: Qualitative Properties of Rulings

Jason	Ku	USA	MIT	126. Filling a Hole in a Crease Pattern: Isometric Mapping of a Polygon given a Folding of its Boundary
Kaori	Kuribayashi-Shigetomi	Japan	Hokkaido University	91. Cell Origami : Producing 3D tissue using origami folding technique
Masahiro	Kushida	Japan	Anan National College of Technology	104. On Quadratic Curves Generated from Quasi-Fish Bases
Shi-Pui	Kwan	Hong Kong	The Hong Kong Institute of Education	64. Mathematics and Art through the Origami of Cuboctahedron
Kelly	Kye	USA	NC State University	65. Polymer Sheets that Self-fold in Response to Light
Robert	Lang	USA	langorigami.com	87. Tessellatica: A Mathematica System for Origami Analysis
				86. A Graphical Technique for Designing Rigidly Foldable Origami Mechanisms
				84. Origami Graph Paper for Polygon Packing and One-Straight-Cut
				150. An Offset Construction Technique for Thick Rigidly Foldable Origami
				83. Master Peace: An Evolution of Monumental Origami
				82. Spiderwebs, Tilings, and Flagstone Tessellations
				85. Pentasia: an Aperiodic Origami Surface
				106. Engineering biological tissues by directed assembly and origami folding
				31. Micro-patterned polymer gel multilayers for reversibly self-folding origami
Peng Yee	Lee	Singapore	National Institute of Education, Singapore	12. Using paper folding to solve problems in school geometry
Cheng Chit	Leong	Singapore	self employed	18. Design of Closed Origami Polyhedral Surface by Straight-crease Couplets
Yew Wei	Leong	Singapore	Institute of Materials Research and Engineering	58. Experiments and Analysis of the Miura-ori Fold Core
Jyh-Ming	Lien	USA	George Mason University	171. Determine Distinct Shapes of Rigid Origami
Sicong	Liu	Singapore	Nanyang Technological University	58. Experiments and Analysis of the Miura-ori Fold Core
Tian	Liu	USA	Northeastern University	106. Engineering biological tissues by directed assembly and origami folding
Ying	Liu	USA	NC State University	65. Polymer Sheets that Self-fold in Response to Light
Carol	Livermore	USA	Northeastern University	106. Engineering biological tissues by directed assembly and origami folding
Guoxing	Lu	Singapore	Nanyang Technological University	58. Experiments and Analysis of the Miura-ori Fold Core
Anna	Lubiw	Canada	University of Waterloo	44. Rigid Flattening of Polyhedra with Slits
Bryce	Lyon	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Jun	Maekawa	Japan	National Astronomical Observatory of Japan	35. Computational Problems Related to Paper Crane in the Edo Period
Spencer	Magleby	USA	Brigham Young University - CMR Lab	140. Creating Novel Motions with N-Long Chains of Interconnected Spherical Mechanisms
				86. A Graphical Technique for Designing Rigidly Foldable Origami Mechanisms
				150. An Offset Construction Technique for Thick Rigidly Foldable Origami
L.	Mahadevan	USA	Harvard University	178. A Simple Mechanical Simulation of Curved and Tessellated Origami Structures
Russell	Mailen	USA	NC State University	65. Polymer Sheets that Self-fold in Response to Light
Richard	Malak	USA	Texas A&M University - Department of Mechanical Engineering	116. Miura-Ori Rectilinear Locomotion Using SMA Actuators
				94. Experimental Analysis of Self-Folding SMA-based Sheets for Origami Engineering
				48. Self-Folding of Reconfigurable Complex Structures using Programmable Active Laminates
Rupert	Maleczek	Austria	Archispass, Innsbruck, Austria	158. Selforganised folding with gravity and friction as guiding concept
				157. Study on Deployable Linear Folded Stripe Structures
Linda	Marlina	Indonesia	Klub Origami Indonesia	24. Origami as Teaching Media for Early Childhood Education in Indonesia (Training for Teachers)

Lauren	McLeod	USA	Cornell University	77. Mechanics of Snap-Through Transitions in Twisted Origami
				76. Mechanics of Miura-ori Lattice Defects
Lainey	Mcquain	USA	Elon University	78. Origami and Teaching Language and Composition
Philipp	Mehner	USA	Northeastern University	106. Engineering biological tissues by directed assembly and origami folding
Brett	Mellor	USA	Brigham Young University	103. The Design and Production of a Folded Felt Stool
Peter	Middendorf	Germany	Institut für Flugzeugbau / Universität Stuttgart	129. Comparison of Mechanical Properties of Periodic Non-flat Tessellations
Jun	Mitani	Japan	University of Tsukuba	73. Simple Flat Origami Exploration System with Random Folds
				119. Designing Nth Order Rotational Symmetry Origami From 4th Order Symmetric Crease Patterns
				27. Self-Intersecting Origami with Cuts
				97. Curved Origami for Developable Surface Coupling
				101. A Study on Orthogonal Pleat Tessellation and Its Folding Sequence
				96. Generating Folding Sequences from Crease Patterns of Flat-Foldable Origami
				72. Flat-Foldable Axisymmetric Structures with Open Edges
Koryo	Miura	Japan	University of Tokyo	143. Three-point method of folding rectangular paper into Miura-ori
Yoshinobu	Miyamoto	Japan	Aichi Institute of Technology	115. Rotational Erection System (RES): Origami Extended with Cuts
David	Morgan	USA	Brigham Young University	103. The Design and Production of a Folded Felt Stool
Hiroaki	Morimura	Japan	Tokyo Institute of Technology	161. Sound Insulating Performance on Origami-based Sandwich Trusscore Panels
Jeannine	Mosely	USA	none	62. Crowdsourcing Origami Sculptures
Daniel	Myers	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Perla	Myers	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Jun-Hee	Na	USA	University of Massachusetts	31. Micro-patterned polymer gel multilayers for reversibly self-folding origami
Chie	Nara	Japan	Tokai University	100. Flattening Polyhedra with Two Adjacent Rigid Faces
Roberto Alcarria Do	Nascimento	Brazil	UNESP	47. Geometric Strategies to Create Origamic Architecture Models
Mark	Neyrinck	USA	Johns Hopkins University	117. Cosmological Origami: Folding up the Dark-Matter Sheet into the Cosmic Web
Taketoshi	Nojima	Japan	Meiji University	155. Origami-based Modeling Techniques for Deployable Meandering Tubes
John	Oberman	Israel	Pre-service Mathematics Training, Shaanan Academic College, Haifa, Israel.	66. Origametria: Research into its Effectiveness as a Tool for Developing Flexibility in Geometric Thinking
Yoji	Okabe	Japan	the University of Tokyo	147. Self-Deploying Origami Models Using Misaligned Crease Patterns
Masao	Okamura	Japan	Advisor, Japan Origami Academic Society	inv-1. Origami works that Kuzuhara Koto brought down to us
Robert	Orndorff	USA	Unaffiliated	121. Hypothesis and Model for Evaluating an Elementary School Origami Program
Chris	Palmer	USA	UC Berkeley	145. Paper and Textile Folding: A Synergistic Relationship
Tawni	Paradise	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Jorge	Pardo	Spain	EMOZ	14. EMOZ (Educational Museum Origami Zaragoza)
Glaucio H.	Paulino	USA	University of Illinois at Urbana Champaign	137. Toward optimization of stiffness and flexibility of rigid, flat-foldable origami structures
Rui	Peng	China	Tianjin University	61. The Metamaterial generated from Rigid-origami Pattern
Edwin A.	Peraza Hernandez	USA	Aerospace Engineering Texas A&M in College Station	116. Miura-Ori Rectilinear Locomotion Using SMA Actuators
				48. Self-Folding of Reconfigurable Complex Structures using Programmable Active Laminates
Leon	Poladian	Australia	University of Sydney	29. Using the Fujimoto Approximation Technique to teach Chaos Theory
Sue	Pope	UK	University of Manchester	52. Origami for connecting mathematical ideas and building relational understanding of mathematics

Aaron	Powledge	USA	Texas A&M University - Department of Mechanical Engineering	94. Experimental Analysis of Self-Folding SMA-based Sheets for Origami Engineering
Kayhan F.	Qaiser	Canada	McGill University	127. Scaling a Surface down to Any Fraction by Twist Folding
Chen	Qiu	UK	King's College London	54. Screw Algebra Based Kinematic and Static Modeling Approach for Origami Enabled Structures
Jennifer	Ramseyer	USA	MIT	173. Weaving a Uniformly Thick Sheet from Rectangles
Abhinav	Rao	USA	University of Michigan	167. Millimeter scale self-folding of a laminated paper system
Gregory	Reich	USA	Air Force Research Laboratory	74. Physics-based Optimization of Origami Structures through FEM
Megan	Roberts	USA	MIT	166. A modular collapsible folded paper tower
José Ignacio	Royo Prieto	Spain	University of the Basque Country UPV/EHU	132. Abelian and non-Abelian numbers via 3D-origami
Alan	Russell	USA	Elon University	78. Origami and Teaching Language and Composition
				65. Polymer Sheets that Self-fold in Response to Light
Kazuya	Saito	Japan	the University of Tokyo	147. Self-Deploying Origami Models Using Misaligned Crease Patterns
Chris	Santangelo	USA	University of Massachusetts Amherst	77. Mechanics of Snap-Through Transitions in Twisted Origami
Christian	Santangelo	USA	University of Massachusetts, Amherst	118. Geometrically controlled catastrophe and the link between snap buckling, origami, and material science
				76. Mechanics of Miura-ori Lattice Defects
				31. Micro-patterned polymer gel multilayers for reversibly self-folding origami
Mariko	Sasakura	Japan	Okayama University Graduate School of Natural Science and Technology	92. Origami on the iPad
Maria	Savchenko	Japan	Meiji University	122. Approach for unfolding 3D meshes for crafting paper models
Ray	Schamp	USA	rayschamp.com	148. Characterization of Origami Corrugation Crease Patterns
Kiumars	Sharif	Iran	University of Tehran	40. Using Origami to Enrich Mathematical Understanding of Fractals
Nivedita	Sharma	USA	University of Massachusetts, Amherst	118. Geometrically controlled catastrophe and the link between snap buckling, origami, and material science
Isao	Shimoyama	Japan	University of Tokyo	162. Magnetic Self-Assembly for Three-Dimensional Microstructures
Matthew	Shlian	USA	University of Michigan	166. A modular collapsible folded paper tower
Katrin	Shumakov	Canada	ORILAND	53. Activating Left and Right Brain Functions with Origami
Yuri	Shumakov	Canada	ORILAND	53. Activating Left and Right Brain Functions with Origami
Jesse	Silverberg	USA	Cornell University	118. Geometrically controlled catastrophe and the link between snap buckling, origami, and material science
				77. Mechanics of Snap-Through Transitions in Twisted Origami
				76. Mechanics of Miura-ori Lattice Defects
Maria Luisa	Spreafico	Italy	Politecnico di Torino	57. Modeling Vaults in Origami: a Bridge between Mathematics and Architecture
				59. Area and Paper Optimization
Kerry	Stanko	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Takamichi	Sushida	Japan	Meiji University	63. Origami for triangular spiral tilings
Tomohiro	Tachi	Japan	University of Tokyo	137. Toward optimization of stiffness and flexibility of rigid, flat-foldable origami structures
				90. Locked Rigid Origami with Multiple Degrees of Freedom
				168. Designing Curved-Crease Tessellations of Lenses: Qualitative Properties of Rulings
				98. Rigid Folding of Periodic Triangulated Origami Tessellations
				44. Rigid Flattening of Polyhedra with Slits
Eduard	Taganap	Philippines	Central Luzon State University	110. A Color Symmetry Approach to the Construction of Crystallographic Flat Origami
Kazuko	Takahashi	Japan	Kwansei Gakuin University	105. Automated Construction and Proving of Knot Fold by Eos System

Hiroyuki	Tanabe	Japan	I.P. SYSTEM INC.	92. Origami on the iPad
Shuang	Tang	Japan	University of Tsukuba	119. Designing Nth Order Rotational Symmetry Origami From 4th Order Symmetric Crease Patterns
Sameh	Tawfick	USA	University of Illinois	166. A modular collapsible folded paper tower
Minoru	Taya	USA	University of Washington	183. Design of the Origami-like hinge line of space deployable structures
Nick	Timpone	USA	PrimaveraPD	9. Examining TIMSS Items through the Lens of Origami
Sunao	Tokura	Japan	Tokura Simulation Research Corporation	139. A study on crash energy absorption ability of lightweight structure by using truss core panel
Daniel	Tomkins	USA	Texas A&M University	135. Planning Folding Motions for Shape-Memory Alloy Sheets
Eulàlia	Tramuns	Spain	Universitat Politècnica de Catalunya	123. Geometric and arithmetic relations concerning origami
				132. Abelian and non-Abelian numbers via 3D-origami
Martin	Trautz	Germany	Chair of Structures and Structural Design at RWTH Aachen University	125. A Systematic Overview of Origami-Based Structures in Technical Applications
				152. Comparison of Design Processes in Architecture and Mechanical Engineering
				153. Demands on an Adapted Design Process for Foldable Structures
Naoko	Tsuji	Japan	Handa Sky Science Museum	143. Three-point method of folding rectangular paper into Miura-ori
Akira	Tsukahara	Japan	the University of Tokyo	147. Self-Deploying Origami Models Using Misaligned Crease Patterns
Naoya	Tsuruta	Japan	University of Tsukuba	73. Simple Flat Origami Exploration System with Random Folds
Arnold	Tubis	USA	Purdue University (retired)	17. Origami-Inspired Deductive Threads in Pre-Geometry
Ryuhei	Uehara	Japan	Japan Advanced Institute of Science and Technology	36. A survey and recent results about common developments of two or more boxes
Thais Regina	Ueno Yamada	Brazil	UNESP	46. Origamic Architecture and Van Hiele Model in the Graduate Classroom of Descriptive Geometry
				47. Geometric Strategies to Create Origamic Architecture Models
Richard	Vaia	USA	Air Force Research Laboratory	74. Physics-based Optimization of Origami Structures through FEM
Jan	van der Woerd	Germany	RWTH Aachen University	124. Oricreate: modeling framework for design and manufacturing of folded plate structures
Veronica	Verplancken	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Anna	Walsh	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami
Hai	Wang	China	Shanghai Jiao Tong University	38. Design of Double-Walled Origami-Core Aircraft Fuselage Shell
Tianyi	Wang	Japan	University of Tsukuba	101. A Study on Orthogonal Pleat Tessellation and Its Folding Sequence
Patsy	Wang-Iverson	USA	Gabriella and Paul Rosenbaum Foundation	9. Examining TIMSS Items through the Lens of Origami
Naohiko	Watanabe	Japan	Gifu National College of Technology	23. Application of rigid-foldability condition to yield line analysis
Guowu	Wei	UK	King's College London	160. Folding Polygons to Deployable Convex Polyhedrons
Wensdy	Whitehead	USA	n/a	163. Shovel Folding: Algorithmic Origami Design of Words and Other Line Drawings
Michael	Winckler	Germany	HGS MathComp, Universität Heidelberg	170. "The next model is for Beginners" First steps in Differential Origami
Zhonghua	Xi	USA	George Mason University	171. Determining Distinct Shapes of Rigid Origami
Yoshikazu	Yamagishi	Japan	Ryukoku University	63. Origami for triangular spiral tilings
Akiko	Yamanashi	Japan	JOAS	113. The Report on an Origami Class at a Computer College
Yang	Yang	Japan	Meiji University	139. A study on crash energy absorption ability of lightweight structure by using truss core panel
Elisabeth	Yeruuldelger	USA	University of San Diego	179. Project Mathigami: Exploring Mathematics Through Origami

Zhong	You	UK	University of Oxford	38. Design of Double-Walled Origami-Core Aircraft Fuselage Shell 25. Improvement and optimisation of non-Miura foldcores under impact loading 33. Thin-Walled Deployable Grid Structures 26. Design and analysis of morphing folded shell structures
Bo	Yu	Japan	Meiji University	122. Approach for unfolding 3D meshes for crafting paper models
Fukami	Yuji	Japan	Osaka Prefecture University	71. General Folding Pattern Solution for Solids based on Polygonal Sections
Ketao	Zhang	UK	King's College London	54. Screw Algebra Based Kinematic and Static Modeling Approach for Origami Enabled Structures
Xilu	Zhao	Japan	Saitama Institute of Technology	139. A study on crash energy absorption ability of lightweight structure by using truss core panel
Xiang	Zhou	China	Shanghai Jiao Tong University	38. Design of Double-Walled Origami-Core Aircraft Fuselage Shell
Yong	Zhu	USA	NC State University	65. Polymer Sheets that Self-fold in Response to Light
Ursula	Zich	Italy	Politecnico di Torino	57. Modeling Vaults in Origami: a Bridge between Mathematics and Architecture